Dawson College: **Linear Algebra**: 201-NYA-05 13

November 11, 2010

Last Name: SOLUTIONS

First Name: __________

Student ID: __________

Quiz 2(B)

Question 1. *(5 marks)* Find the first and sectond derivative of the function:

$$f(u) = \frac{u}{u^2+3} \qquad f'(u) = \frac{(1)(u^2+3) - u(2u)}{(u^2+3)^2} = \frac{-u^2+3}{(u^2+3)^2}$$

$$f'(u) = \frac{(-2u)(u^2+3)^2 - (-u^2+3)\left[2(u^2+3)(2u)\right]}{(u^2+3)^4}$$

$$= \frac{2u(u^2+3)\left[-(u^2+3) - 2(-u^2+3)\right]}{(u^2+3)^4}$$

$$= \frac{2u(u^2\cancel{+3})\left[u^2-9\right]}{(u^2+3)^{\cancel{4}\,3}} = \frac{2u(u+3)(u-3)}{(u^2+3)^3}$$

Question 2. *(5 marks)* Find $\frac{dy}{dx}$:

$$x^2y^{1/2} = x^2 + 2y^2$$

$$\frac{d}{dx}(x^2y^{1/2}) = \frac{d}{dx}(x^2) + \frac{d}{dx}(2y^2)$$

$$2xy^{1/2} + x^2\left(\tfrac{1}{2}y^{-1/2}y'\right) = 2x + 4yy'$$

$$\frac{x^2}{2y^{1/2}}y' - 4yy' = 2x - 2xy^{1/2}$$

$$y'\left(\frac{x^2}{2y^{1/2}} - 4y\right) = 2x - 2xy^{1/2}$$

$$y'\left(\frac{x^2 - 8y^{3/2}}{2y^{1/2}}\right) = 2x - 2xy^{1/2}$$

$$y = \frac{(2x - 2xy^{1/2})(2y^{1/2})}{x^2 - 8y^{3/2}} = \frac{4xy^{1/2} - 4xy}{x^2 - 8y^{3/2}}$$

Question 3. *(5 marks)* Find the intervals where the function is increasing, the intervals where the function is decreasing and any relative extrema:

$$f(x) = \frac{6}{3}x^3 - 6x^2 - 18x - 6$$

$f'(x) = 6x^2 - 12x - 18$

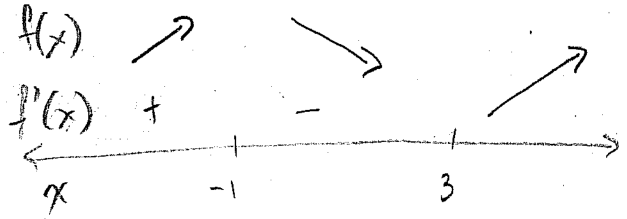
$= 6(x^2 - 2x - 3)$

$= 6(x-3)(x+1)$

$f'(x) = 0 \Leftrightarrow x = -1, 3$

$f'(x)$ EXISTS EVERYWHERE

$\therefore$ THE CRITICAL NUMBERS ARE $x = -1, 3$



TEST POINTS:

$x = -2 \quad f'(-2) = 6(-2)^2 - 12(-2) - 18 = 30 > 0$

$x = 0 \quad f'(0) = -18 < 0$

$x = 4 \quad f'(4) = 6(4)^2 - 12(4) - 18 = 30 > 0$

$\therefore$ f IS INCREASING ON $(-\infty, -1)$ AND $(3, \infty)$

f IS DECREASING ON $(-1, 3)$

$f(-1) = \frac{6}{3}(-1)^3 - 6(-1)^2 - 18(-1) - 6 = 4$ IS A RELATIVE MAXIMUM

$f(3) = \frac{6}{3}(3)^3 - 6(3)^2 - 18(3) - 6 = -60$ IS A RELATIVE MINIMUM