

Last Name: SOLUTIONS

First Name: _____

Student ID: _____

Test 1

The length of the test is 1hr and 45min. You may use a nonprogrammable, nongraphing scientific calculator. Remember to clearly show all of your work and to use correct notation.

Question 1. (4 marks)

Simplify the following expressing your final answer with positive exponents only:

$$\left(\frac{3a^{-2}}{c}\right) \cdot \left(\frac{a^2b^{-2}}{16a^6b^8c^{-3}}\right)^{-1/2} = \frac{3}{a^2c} \cdot \frac{(a^2)^{-1/2} (b^{-2})^{-1/2}}{(16)^{-1/2} (a^6)^{-1/2} (b^8)^{-1/2} (c^{-3})^{-1/2}}$$

$$= \frac{3}{a^2c} \cdot \frac{a^{-1} b^1}{16^{-1/2} a^{-3} b^{-4} c^{3/2}} = \frac{3 \cdot 16^{1/2} a^3 b^4 b}{a^2 c a c^{3/2}} = \frac{3 \cdot 4 a^3 b^5}{a^3 c^{5/2}}$$

$$= \frac{12 b^5}{c^{5/2}}$$

Question 2. (4 marks) Simplify the following expressing your final answer as a single rational function with positive exponents only:

$$\frac{(x-2)(3x+2)^{-1/2} + (3x+2)^{1/2}}{x^2} = \frac{\frac{x-2}{(3x+2)^{1/2}} + (3x+2)^{1/2}}{x^2}$$

$$= \frac{\frac{x-2}{(3x+2)^{1/2}} + \frac{(3x+2)^{1/2} (3x+2)^{1/2}}{(3x+2)^{1/2}}}{x^2} = \frac{(x-2) + (3x+2)}{(3x+2)^{1/2} x^2}$$

$$= \frac{4x}{(3x+2)^{1/2} x^2} = \frac{4}{x(3x+2)^{1/2}}$$

Question 3. (5 marks) A company has determined that for a certain product the supply equation is $2p = 4x + 36$ and the demand equation is $p = -2x^2 - x + 32$ where x represents the quantity demanded in units of a thousand and p is the unit price in dollars. Find the equilibrium price and equilibrium quantity for the product.

$$2p = 4x + 36 \Rightarrow p = 2x + 18$$

INTERSECTION $-2x^2 - x + 32 = 2x + 18$

$$0 = 2x^2 + 3x - 14$$

$$0 = 2x^2 + 7x - 4x - 14$$

$$0 = x(2x + 7) - 2(2x + 7)$$

$$0 = (x - 2)(2x + 7)$$

$$\therefore x = 2 \quad \text{or} \quad x = \cancel{-\frac{7}{2}}$$

$$\alpha \cdot \beta = -28$$

$$\alpha + \beta = 3$$

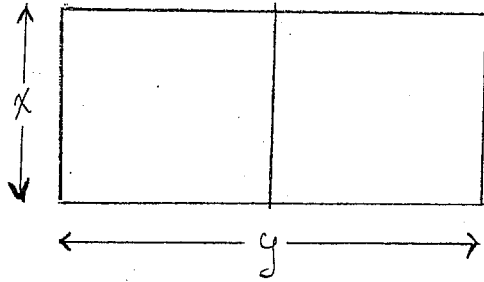
$$\alpha = 7 \quad \beta = -4$$

∴ THE EQUILIBRIUM QUANTITY IS 2000

$$p = 2(2) + 18 = 22$$

∴ THE EQUILIBRIUM PRICE IS \$22.

Question 4. (5 marks) A farmer wants to build a fence according to the diagram below. She has 300m of fencing with which to build the fence. Express the area contained within the fence as a function of x . What is the domain of this function?



$$A = xy$$

$$P = 3x + 2y = 300$$

$$\Rightarrow 2y = 300 - 3x$$

$$y = 150 - \frac{3}{2}x$$

$$\therefore A(x) = x \left(150 - \frac{3}{2}x \right)$$

$$A(x) = 150x - \frac{3}{2}x^2$$

WE NOTE THAT $x \geq 0$ AND ALSO $3x \leq 300 \Rightarrow x \leq 100$

SO THE DOMAIN OF $A(x)$ IS $[0, 100]$.

Question 5. (10 marks) Evaluate the following limits:

$$(a) \lim_{x \rightarrow -3} \frac{x^2 - 4x - 21}{x^2 - 9} \quad \underline{\underline{\text{I.F. } \frac{0}{0}}} \quad \lim_{x \rightarrow -3} \frac{(x-7)(x+3)}{(x+3)(x-3)}$$

$$= \lim_{x \rightarrow -3} \frac{x-7}{x-3} = \frac{-3-7}{-3-3} = \frac{-10}{-6} = \frac{5}{3}$$

$$(b) \lim_{x \rightarrow 5} \frac{x^2 - 4x - 5}{x^2 + x + 7} = \frac{(5)^2 - 4(5) - 5}{(5)^2 + 5 + 7} = \frac{0}{37} = 0$$

$$(c) \lim_{x \rightarrow 9} \frac{\sqrt{x} - 3}{x - 9} \quad \underline{\underline{\text{I.F. } \frac{0}{0}}} \quad \lim_{x \rightarrow 9} \frac{\sqrt{x} - 3}{x - 9} \cdot \frac{\sqrt{x} + 3}{\sqrt{x} + 3}$$

$$= \lim_{x \rightarrow 9} \frac{x - 3\sqrt{x} + 3\sqrt{x} - 9}{(x-9)(\sqrt{x}+3)} = \lim_{x \rightarrow 9} \frac{x-9}{(x-9)(\sqrt{x}+3)}$$

$$= \lim_{x \rightarrow 9} \frac{1}{\sqrt{x}+3} = \frac{1}{\sqrt{9}+3} = \frac{1}{6}$$

$$(d) \lim_{x \rightarrow \infty} \frac{-3x^3 + 2x - 1}{4x^3 - 2x^2 + 5} = \lim_{x \rightarrow \infty} \frac{\frac{-3x^3}{x^3} + \frac{2x}{x^3} - \frac{1}{x^3}}{\frac{4x^3}{x^3} - \frac{2x^2}{x^3} + 5}$$

$$= \lim_{x \rightarrow \infty} \frac{-3 + 2/x^2 - 1/x^3}{4 - 2/x + 5/x^3} = \frac{-3 + 0 - 0}{4 - 0 + 0} = -\frac{3}{4}$$

$$(e) \lim_{x \rightarrow -\infty} \frac{5x^2 - 6x + 3}{x^3 - 4x + 2} = \lim_{x \rightarrow -\infty} \frac{\frac{5x^2}{x^3} - \frac{6x}{x^3} + \frac{3}{x^3}}{\frac{x^3}{x^3} - \frac{4x}{x^3} + \frac{2}{x^3}}$$

$$= \lim_{x \rightarrow -\infty} \frac{\frac{5}{x} - \frac{6}{x^2} - \frac{3}{x^3}}{1 - 4/x^2 + \frac{2}{x^3}} = \frac{0 - 0 - 0}{1 - 0 + 0} = 0$$

Question 6. (5 marks) Find the values of x for which the following function is continuous (explain your reasoning):

$$f(x) = \begin{cases} x^2 - 3x + 3 & \text{if } x < 2 \\ 1 & \text{if } x = 2 \\ 5x - 9 & \text{if } x > 2 \end{cases}$$

WE KNOW THAT f IS CONTINUOUS ON $(-\infty, 2) \cup (2, \infty)$

SINCE IT IS A POLYNOMIAL THERE

WE NEED TO CHECK THESE 3 CONDITIONS FOR CONTINUITY FOR $x = 2$.

1) $f(2) = 1$ so $f(2)$ IS DEFINED

2) $\lim_{x \rightarrow 2^-} f(x) = \lim_{x \rightarrow 2^-} x^2 - 3x + 3 = (2)^2 - 3(2) + 3 = 1$

$\lim_{x \rightarrow 2^+} f(x) = \lim_{x \rightarrow 2^+} 5x - 9 = 5(2) - 9 = 1$

$\therefore \lim_{x \rightarrow 2} f(x)$ EXISTS SINCE $\lim_{x \rightarrow 2^-} f(x) = \lim_{x \rightarrow 2^+} f(x)$

3) $f(2) = 1 = \lim_{x \rightarrow 2} f(x)$

$\therefore f$ IS CONTINUOUS AT $x = 2$

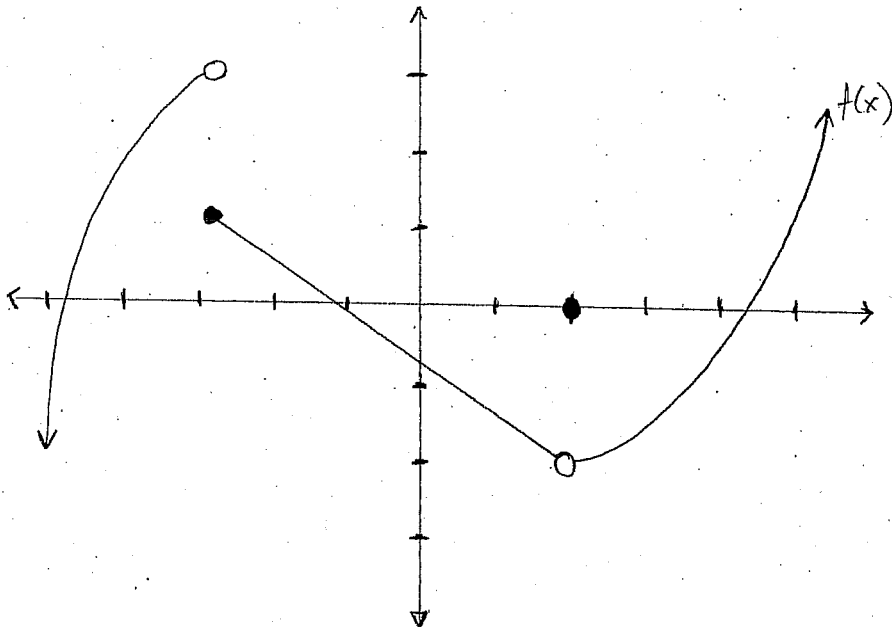
$\therefore f$ IS CONTINUOUS ON $(-\infty, \infty)$

Question 7. (6 marks) Given the following graph, find the following if it exists:

(a) $\lim_{x \rightarrow -3^-} f(x)$, $\lim_{x \rightarrow -3^+} f(x)$, $\lim_{x \rightarrow -3} f(x)$

(b) $\lim_{x \rightarrow 2} f(x)$, $f(2)$

(c) Is $f(x)$ continuous at $x = 2$? Why or why not?



a) $\lim_{x \rightarrow -3^-} f(x) = 3$

$\lim_{x \rightarrow -3^+} f(x) = 1$

$\lim_{x \rightarrow -3} f(x)$ DOES NOT EXIST

b) $\lim_{x \rightarrow 2} f(x) = -2$, $f(2) = 0$

c) f IS NOT CONTINUOUS AT $x=2$ SINCE

$\lim_{x \rightarrow 2} f(x) \neq f(2)$ (FAILS CONDITION 3)

Bonus Question. (3 marks) Find the values of a and b that make the following function continuous on all real numbers.

$$f(x) = \begin{cases} 2ax+b & \text{if } x < 1 \\ 2bx^2+4 & \text{if } 1 \leq x \leq 2 \\ ax-14 & \text{if } x > 2 \end{cases}$$

f IS CONTINUOUS EVERYWHERE EXCEPT POSSIBLY AT $x=1$ AND $x=2$

AT $x=1$

$$1) f(1) = 2b(1)^2 + 4 = 2b + 4$$

$$2) \lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^-} 2ax + b = 2a(1) + b = 2a + b$$

$$\lim_{x \rightarrow 1^+} f(x) = \lim_{x \rightarrow 1^+} 2bx^2 + 4 = 2b(1)^2 + 4 = 2b + 4$$

\therefore WE NEED $2a + b = 2b + 4$ FOR $\lim_{x \rightarrow 1} f(x)$ TO EXIST.

$$3) \text{ AGAIN WE WANT } \underbrace{2a + b}_{\lim_{x \rightarrow 1} f(x)} = \underbrace{2b + 4}_{f(1)}$$

\therefore IF $2a + b = 2b + 4$ f IS CONTINUOUS AT $x=1$

$$\Rightarrow 2a - b = 4$$

AT $x=2$

$$1) f(2) = 2b(2)^2 + 4 = 8b + 4$$

$$2) \lim_{x \rightarrow 2^-} f(x) = \lim_{x \rightarrow 2^-} 2bx^2 + 4 = 2b(2)^2 + 4 = 8b + 4$$

$$\lim_{x \rightarrow 2^+} f(x) = \lim_{x \rightarrow 2^+} ax - 14 = a(2) - 14 = 2a - 14$$

\therefore WE NEED $2a - 14 = 8b + 4$ FOR $\lim_{x \rightarrow 2} f(x)$ TO EXIST

$$3) \text{ AGAIN WE NEED } \underbrace{2a - 14}_{\lim_{x \rightarrow 2} f(x)} = \underbrace{8b + 4}_{f(2)}$$

\therefore IF $2a - 14 = 8b + 4$ f IS CONTINUOUS AT $x=2$

$$\Rightarrow 8b - 2a = -18$$

$$\text{now } 2a - b = 4 \Rightarrow 2a = b + 4$$

$$8b - 2a = -18 \Rightarrow 8b - (b + 4) = -18$$

$$7b - 4 = -18$$

$$7b = -14$$

$$b = -2$$

$$\therefore 2a = b + 4$$

$$= -2 + 4$$

$$= 2$$

$$a = 1$$

\therefore IF $a=1$ AND $b=-2$ f IS CONTINUOUS
ON ALL REAL NUMBERS.