**Dawson College: Calculus I:** 201-NYA-05 13

October 29, 2010

**Last Name:** SOLUTIONS

**First Name:** ____________

**Student ID:** ____________

# Test 2A

The length of the test is 1hr and 45min. You may use a nonprogrammable, nongraphing scientific calculator. Remember to clearly show all of your work and to use **correct notation**.

**Question 1.** *(5 marks)* Use the (limit) definition of the dervative to find the derivative of $f(x) = 2x - x^2$.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{[2(x+h) - (x+h)^2] - [2x - x^2]}{h}$$

$$= \lim_{h \to 0} \frac{[2x + 2h - (x^2 + 2xh + h^2)] - 2x + x^2}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{2x} + 2h - \cancel{x^2} - 2xh - h^2 - \cancel{2x} + \cancel{x^2}}{h}$$

$$= \lim_{h \to 0} \frac{2h - 2xh - h^2}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(2 - 2x - h)}{\cancel{h}}$$

$$= \lim_{h \to 0} (2 - 2x - h)$$

$$= 2 - 2x - 0$$

$$= 2 - 2x$$

**Question 2.** *(11 marks)* Find the derivatives of the following functions. **Do not simplify your answer.**

(a) $f(x) = \frac{4}{x^3} + \frac{2}{\sqrt{x}} - \sqrt[3]{x^2}$ $= 4x^{-3} + 2x^{-1/2} - x^{2/3}$

$$= -12x^{-4} - x^{-3/2} - \frac{2}{3}x^{-1/3}$$

(b) $g(x) = \left(\frac{2x+1}{x-2}\right)^{3/2}$

$$g'(x) = \frac{3}{2}\left(\frac{2x+1}{x-2}\right)^{1/2} \frac{d}{dx}\left(\frac{2x+1}{x-2}\right)$$

$$= \frac{3}{2}\left(\frac{2x+1}{x-2}\right)^{1/2} \frac{2(x-2)-(2x+1)(1)}{(x-2)^2}$$

**(c)** $F(x) = (5x^3 - x^2 + 2)\left(x^2 + 3x - \frac{2}{x}\right)$

$$F'(x) = \frac{d}{dx}\left[5x^3 - x^2 + 2\right] \cdot \left(x^2 + 3x - \frac{2}{x}\right) + (5x^3 - x^2 + 2)\frac{d}{dx}\left(x^2 + 3x - \frac{2}{x}\right)$$

$$= (15x^2 - 2x)\left(x^2 + 3x - \frac{2}{x}\right) + (5x^3 - x^2 + 2)\left(2x + 3 + \frac{2}{x^2}\right)$$

**Question 3.** *(4 marks)* Find all $x$ values where the tangent line to

$$f(x) = \frac{2}{3}x^3 + \frac{9}{2}x^2 - 5x + 10$$

is horizontal.

$$f'(x) = 2x^2 + 9x - 5 = 0$$

$$2x^2 + 10x - x - 5 = 0$$

$$2x(x+5) - (x+5) = 0$$

$$(2x-1)(x+5) = 0$$

$$x = -5,\ \tfrac{1}{2}$$

**Question 4.** *(3 marks)* Let

$$h(x) = \frac{f(x)}{g(x) - x}$$

Given $f(2) = 1$, $g(2) = -2$, $f'(2) = 2$, and $g'(2) = 4$, find $h'(2)$

$$h'(x) = \frac{f'(x)[g(x) - x] - f(x)[g'(x) - 1]}{[g(x) - x]^2}$$

$$h'(2) = \frac{f'(2)[g(2) - 2] - f(2)[g'(2) - 1]}{[g(2) - 2]^2}$$

$$= \frac{2[-2-2] - (1)[4-1]}{(-2-2)^2}$$

$$= \frac{-8-3}{16} = -\frac{11}{16}$$

**Question 5. (a)**
*(4 marks)* The following is a problem from a calculus test that I took in University and my solution which is incorrect. Find and expain the four main mistakes that I made (only four, not bad!). (Note: full marks will not be given for only finding the mistakes without sufficient explanation). You will be asked to solve the problem in part **(b)**.

**Problem:** Find an equation of the tangent line to the graph of $f(x) = (2x-1)^2(x^2-x+4)$ at $(1,4)$.

**"Solution":**

NEED TO USE PRODUCT RULE

$$\begin{aligned} f'(x) &= \frac{d}{dx}\left[(2x-1)^2\right]\cdot\frac{d}{dx}(x^2-x+4) \\ &= 2(2x-1)(2)\cdot(2x-0) \\ &= 4(2x-1)(2x) \end{aligned}$$

NOT THE DERIVATIVE OF $x^2-x+2$ SINCE $\frac{d}{dx}(x) = 1$ NOT 0.

$$\begin{aligned} f'(4) &= 4[2(4)-1][2(4)] \\ &= 4(7)(8) \\ &= 224 \end{aligned}$$

USED $y=4$ INSTEAD OF $x=1$

So the slope of the tangent line is $a = 224$.

Tangent line:

$$\begin{aligned} y &= mx+b \\ 1 &= 224(4)+b \\ -895 &= b \end{aligned}$$

SWITCHED $x$ AND $y$.

Therefore the equation of the tangent line is:

$$y = 224x - 895$$

**(b)** *(5 marks)* Write a correct solution for the problem in part **(a)**.

$$f(x) = (2x-1)^2(x^2-x+4)$$

$$f'(x) = 2(2x-1)\cdot(2)(x^2-x+4) + (2x-1)^2(2x-1)$$

$$f'(1) = 4[2(1)-1][(1)^2-(1)+4] + [2(1)-1]^2[2(1)-1]$$

$$= 4(1)(4) + (1)^2(1)$$

$$= 17 \leftarrow \text{SLOPE OF THE TANGENT LINE}$$

$$y = mx + b$$

$$4 = 17(1) + b$$

$$-13 = b$$

$$\therefore \boxed{y = 17x - 13}$$

**Question 6.** *(6 marks)* The weekly demand function for a certain product is

$$p = -0.07x + 700$$

where $p$ denotes the wholesale price in dollars and $x$ denotes the quantity demanded. The weekly total cost function asociated witht manufacturing the product is given by

$$C(x) = 0.0000003x^3 - 0.02x^2 + 400x + 70000$$

where $c(x)$ denotes the total cost incurred in producing $x$ units of the product.

**(a)** Find the revenue function $R(x)$ and the profit funtion $P(x)$.

$$R(x) = xp = x(-0.07x + 700) = -0.07x^2 + 700x$$

$$P(x) = R(x) - C(x)$$

$$= -(0.0000003x^3 - 0.02x^2 + 400x + 70000) + (-0.07x^2 + 700x)$$

$$= -0.0000003x^3 - 0.05x^2 + 300x - 70\,000$$

**(b)** Find the marginal cost function. (only!)

$$c'(x) = 0.0000009x^2 - 0.04x + 400$$

**(c)** Compute $C'(1800)$. What does this value tell us?

$$c'(1800) = 0.0000009(1800)^2 - 0.04(1800) + 400$$

$$= 330.916$$

THIS VALUE TELLS US THAT THE COST OF PRODUCING THE 1801th UNIT IS APPROXIMATELY \$ 330.92

**Question 7.** *(5 marks)* The weekly demand function for a certain product is

$$p = -0.05x + 800$$

where $p$ denotes the wholesale price in dollars and $x$ denotes the quantity demanded.

**(a)** Find the elasticity of demand $E(p)$.

$p = -0.05x + 800$

$0.05x = -p + 800$

$x = -20p + 16000 = f(p)$

$f'(p) = -20$

$$E(p) = -\frac{p f'(p)}{f(p)} = -\frac{p(-20)}{-20p + 16000} = \frac{\cancel{20}p}{\cancel{20}(800 - p)}$$

$$= \frac{p}{800 - p}$$

**(b)** Is demand elastic, inelastic or unitary when price is \$500? What happens to revenue if the price is raised slightly from \$500?

$$E(500) = \frac{500}{800-500} = \frac{5}{3} > 1$$

$\therefore$ ELASTICITY IS ELASTIC AT \$500

IF PRICED IS RAISED SLIGHTRY FROM \$500 THE REVENUE WILL DECREASE.