Dawson College: Calculus I: 201-NYA-05 13

October 29, 2010

Last Name: SOLUTIONS

First Name: ____________

Student ID: ____________

Test 2B

The length of the test is 1hr and 45min. You may use a nonprogrammable, nongraphing scientific calculator. Remember to clearly show all of your work and to use **correct notation**.

Question 1. *(5 marks)* Use the (limit) definition of the dervative to find the derivative of $f(x) = 3x - x^2$.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{[3(x+h) - (x+h)^2] - [3x - x^2]}{h}$$

$$= \lim_{h \to 0} \frac{3x + 3h - (x^2 + 2xh + h^2) - 3x + x^2}{h}$$

$$= \lim_{h \to 0} \frac{3h - 2xh - h^2}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(3 - 2x - h)}{\cancel{h}}$$

$$= \lim_{h \to 0} (3 - 2x - h)$$

$$= 3 - 2x - 0$$

$$= 3 - 2x$$

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Question 2. (*11 marks)* Find the derivatives of the following functions. **Do not simplify your answer.**

(a) $f(x) = \frac{3}{x^4} + \frac{4}{\sqrt{x}} - \sqrt[3]{x^2}$ $= 3x^{-4} + 4x^{-1/2} - x^{2/3}$

$f'(x) = -12x^{-5} - 2x^{-3/2} - \frac{2}{3}x^{-1/3}$

(b) $g(x) = (4x^3 - 2x^2 + 1)\left(x^2 + 2x - \frac{2}{x}\right)$

$g'(x) = \frac{d}{dx}(4x^3 - 2x^2 + 1) \cdot (x^2 + 2x - \frac{2}{x}) + (4x^3 - 2x^2 + 1) \frac{d}{dx}(x^2 + 2x - \frac{2}{x})$

$= (12x^2 - 4x)(x^2 + 2x - \frac{2}{x}) + (4x^3 - 2x^2 + 1)(2x + 2 + \frac{2}{x^2})$

(c) $h(x) = \left(\dfrac{2x+2}{x-3}\right)^{3/2}$

$$h'(x) = \frac{3}{2}\left(\frac{2x+2}{x-3}\right)^{1/2} \frac{d}{dx}\left(\frac{2x+2}{x-3}\right)$$

$$= \frac{3}{2}\left(\frac{2x+2}{x-3}\right)^{1/2} \frac{2(x-3) - (2x+2)(1)}{(x-3)^2}$$

Question 3. *(3marks)* Let

$$h(x) = \frac{g(x)}{f(x) - x}$$

Given $f(3) = 2$, $g(3) = -1$, $f'(3) = 3$, and $g'(3) = 5$, find $h'(3)$

$$h'(x) = \frac{g'(x)[f(x) - x] - g(x)[f'(x) - 1]}{[f(x) - x]^2}$$

$$h'(3) = \frac{g'(3)[f(3) - 3] - g(3)[f'(3) - 1]}{[f(3) - 3]^2}$$

$$= \frac{5[2-3] - (-1)[3-1]}{[2-3]^2}$$

$$= \frac{-5+2}{(-1)^2} = -3$$

Question 4. *(4 marks)* Find all x values where the tangent line to

$$f(x) = \frac{4}{3}x^3 + \frac{15}{2}x^2 - 4x + 2$$

is horizontal.

$f'(x) = 4x^2 + 15x - 4 = 0$

$4x^2 + 16x - x - 4 = 0$

$4x(x+4) - (x+4) = 0$

$(4x-1)(x+4) = 0$

$x = -4, 1/4$

Question 5.

(a) *(4 marks)* The following is a problem from a calculus test that I took in University and my solution which is incorrect. Find and expain the four main mistakes that I made (only four, not bad!). (Note: full marks will not be given for only finding the mistakes without sufficient explanation) You will be asked to solve the problem in part **(b)**.

Problem: Find an equation of the tangent line to the graph of $f(x) = (3x-2)^2(x^2-x+2)$ at $(1,2)$.

"Solution":

$$\begin{aligned} f'(x) &= \tfrac{d}{dx}\left[(3x-2)^2\right]\cdot\tfrac{d}{dx}(x^2-x+2) \leftarrow \text{DIDN'T USE PRODUCT RULE} \\ &= 2(3x-2)(3)\cdot(2x-0) \leftarrow \tfrac{d}{dx}(x) = 1 \text{ NOT } 0. \\ &= 6(3x-2)(2x) \end{aligned}$$

USED $y=2$ INSTEAD OF $x=1$

$$\begin{aligned} f'(2) &= 6[3(2)-2][2(2)] \\ &= 6(4)(4) \\ &= 96 \end{aligned}$$

So the slope of the tangent line is $a = 96$

Tangent line:

$$\begin{aligned} y &= mx+b \\ 1 &= 96(2)+b \leftarrow \text{SWITCHED } x \text{ AND } y. \\ -191 &= b \end{aligned}$$

Therefore the equation of the tangent line is:

$$y = 96x - 191$$

(b) *(5 marks)* Write a correct solution for the problem in part **(a)**.

$f(x) = (3x-2)^2(x^2-x+2)$

$f'(x) = 2(3x-2)(3)(x^2-x+2) + (3x-2)^2(2x-1)$

$f'(1) = 6(3-2)(1-1+2) + (3-2)^2(2-1)$

$= 6(1)(2) + (1)(1)$

$= 13.$ ← SLOPE OF TANGENT LINE

$y = mx+b$

$2 = 13(1)+b$

$-11 = b$

$\therefore \boxed{y = 13x - 11}$

Question 6. *(6 marks)* The weekly demand function for a certain product is

$$p = -0.06x + 800$$

where p denotes the wholesale price in dollars and x denotes the quantity demanded. The weekly total cost function asociated witht manufacturing the product is given by

$$C(x) = 0.0000002x^3 - 0.01x^2 + 300x + 60000$$

where $c(x)$ denotes the total cost incurred in producing x units of the product.

(a) Find the revenue function $R(x)$ and the profit funtion $P(x)$.

$R(x) = xp = x(-0.06x + 800) = -0.06x^2 + 800x$

$P(x) = R(x) - C(x)$

$= (-0.06x^2 + 800x) - (0.0000002x^3 - 0.01x^2 + 300x + 60000)$

$= -0.0000002x^3 - 0.05x^2 + 500x - 60000$

(b) Find the marginal cost function. (only!)

$C'(x) = 0.0000006x^2 - 0.02x + 300$

(c) Compute $C'(1900)$. What does this value tell us?

$C'(1900) = 0.0000006(1900)^2 - 0.02(1900) + 300$

$= 264.166$

$\therefore$ THIS VALUE TELLS US THAT THE COST OF PRODUCING THE 1901th IS APPROXIMATELY \$264.17.

Question 7. *(5 marks)* The weekly demand function for a certain product is

$$p = -0.04x + 600$$

where p denotes the wholesale price in dollars and x denotes the quantity demanded.

(a) Find the elasticity of demand $E(p)$.

$$P = -0.04x + 600$$

$$0.04x = -p + 600$$

$$x = -25p + 15\,000 = f(p)$$

$$f'(p) = -25$$

$$\therefore E(p) = -\frac{p\,f'(p)}{f(p)} = \frac{-p(-25)}{-25p + 15\,000}$$

$$= \frac{25p}{-25p + 15000} = \frac{25p}{25(-p + 600)}$$

$$= \frac{p}{600 - p}$$

(b) Is demand elastic, inelastic or unitary when price is \$400? What happens to revenue if the price is raised slightly from \$400?

$$E(400) = \frac{400}{600 - 400} = 2 > 1$$

$\therefore$ DEMAND IS ELASTIC AT \$400

IF PRICE IS RAISED SLIGHTRY FROM \$400 THE REVENUE WILL DECREASE.