

Last Name: SOLUTION

First Name: _____

Student ID: _____

Quiz 1 (B)

Question 1. (3 marks) Simplify the expression (express your answer with positive exponents only):

$$\left(\frac{y^{-3}}{z^{-2}}\right)^2 \left(\frac{z}{y}\right)^4 = \frac{y^{-6}}{z^{-4}} \cdot \frac{z^4}{y^4} = \frac{z^4 z^4}{y^6 y^4} = \frac{z^8}{y^{10}}$$

Question 2. (3 marks) Rationalize the denominator in the expression:

$$\sqrt[3]{\frac{2a}{3b}} = \frac{(2a)^{1/3}}{(3b)^{1/3}} = \frac{(2a)^{1/3}}{(3b)^{1/3}} \cdot \frac{(3b)^{2/3}}{(3b)^{2/3}} =$$

$$= \frac{\sqrt[3]{2a \cdot 3^2 b^2}}{3b} = \frac{\sqrt[3]{18ab^2}}{3b}$$

Question 3. (6 marks) Factor the following completely:

$$\begin{aligned} \text{(a) } & x^3 - 3x^2 - 4x + 12 \\ &= x^2(x-3) - 4(x-3) \\ &= (x-3)(x^2-4) \\ &= (x-3)(x+2)(x-2) \end{aligned}$$

$$\begin{aligned}
 & \text{(b) } x^6 - 64 \\
 & = (x^3)^2 - 8^2 = (x^3 - 8)(x^3 + 8) \\
 & = (x - 2)(x^2 + 2x + 4)(x + 2)(x^2 - 2x + 4)
 \end{aligned}$$

$$\begin{aligned}
 & \text{(c) } 7x^2 - 27x - 4 \\
 & = 7x^2 - 28x + x - 4 \\
 & = 7x(x - 4) + (x - 4) \\
 & = (7x + 1)(x - 4)
 \end{aligned}$$

$$\begin{aligned}
 a \cdot \beta &= -28 \\
 a + \beta &= -27 \\
 a &= -28 \quad \beta = 1
 \end{aligned}$$

Question 4. (8 marks) Simplify the following expressions:

$$\text{(a) } \frac{1}{1 - \frac{1}{1+y}} + \frac{1}{1 - \frac{1}{1-y}} = \frac{1}{\frac{1+y}{1+y} - \frac{1}{1+y}} + \frac{1}{\frac{1-y}{1-y} - \frac{1}{1-y}}$$

$$= \frac{1}{\frac{y}{1+y}} + \frac{1}{\frac{-y}{1-y}} = \frac{1+y}{y} + \frac{1-y}{-y} = \frac{1+y}{y} - \frac{1-y}{y}$$

$$= \frac{(1+y) - (1-y)}{y} = \frac{2y}{y} = 2$$

$$\text{(b) } \frac{a - 9a^{-1}}{a + 4 + 3a^{-1}} = \frac{a - \frac{9}{a}}{a + 4 + \frac{3}{a}} = \frac{\frac{a^2}{a} - \frac{9}{a}}{\frac{a^2}{a} + \frac{4a}{a} + \frac{3}{a}} = \frac{\frac{a^2 - 9}{a}}{\frac{a^2 + 4a + 3}{a}}$$

$$= \frac{a^2 - 9}{a} \cdot \frac{a}{a^2 + 4a + 3} = \frac{(a+3)(a-3)}{(a+3)(a+1)}$$

$$= \frac{a-3}{a+1}$$