

Last Name: SOLUTION

First Name: _____

Student ID: _____

Quiz 2 (A)

Question 1. (5 marks) Find the first and second derivative of the function:

$$f(t) = \frac{t^2 - 1}{t + 1} \quad f'(t) = \frac{2t(t+1) - (t^2 - 1)(1)}{(t+1)^2} = \frac{t^2 + 2t + 1}{(t+1)^2}$$

$$f''(t) = \frac{(2t + 2)(t+1)^2 - (t^2 + 2t + 1)[2(t+1)(1)]}{(t+1)^4}$$

$$= \frac{(t+1)[(2t+2)(t+1) - 2(t^2 + 2t + 1)]}{(t+1)^4}$$

$$= \frac{(t+1)[2t^2 + 4t + 1 - 2t^2 - 4t - 1]}{(t+1)^4} = 0$$

Question 2. (5 marks) Find $\frac{dy}{dx}$:

$$4x + y^2 = \sqrt{xy}$$

$$\frac{d}{dx}(4x) + \frac{d}{dx}(y^2) = \frac{d}{dx}(xy)^{1/2} \quad \text{CHAINS RULE}$$

$$4 + 2yy' = \frac{1}{2}(xy)^{-1/2} [y + xy']$$

$$4 + 2yy' = \frac{y + xy'}{2(xy)^{1/2}}$$

$$8(xy)^{1/2} + 4x^{1/2}y^{3/2}y' = y + xy'$$

$$4x^{1/2}y^{3/2}y' - xy' = y - 8(xy)^{1/2}$$

$$y'(4x^{1/2}y^{3/2} - x) = y - 8(xy)^{1/2}$$

$$y' = \frac{y - 8(xy)^{1/2}}{4x^{1/2}y^{3/2} - x}$$

Question 3. (5 marks) Find the intervals where the function is increasing, the intervals where the function is decreasing and any relative extrema:

$$f(x) = \frac{6}{3}x^3 - 6x^2 - 18x - 6$$

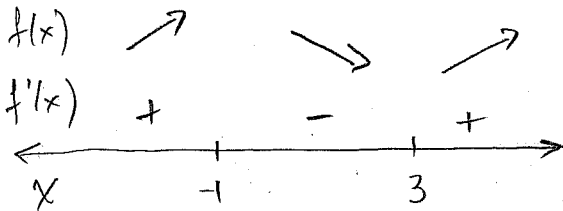
$$f'(x) = 6x^2 - 12x - 18$$

$f'(x)$ IS DEFINED

$$= 6(x^2 - 2x - 3)$$

$$= 6(x-3)(x+1)$$

$$\therefore x = -1, 3$$



TEST POINTS

$$x = -2: f'(-2) = 6(-2)^2 - 12(-2) - 18 = 30 > 0$$

$$x = 0: f'(0) = -18 < 0$$

$$x = 4: f'(4) = 6(4)^2 - 12(4) - 18 = 30 > 0$$

$\therefore f$ IS INCREASING ON $(-\infty, -1)$ AND $(3, \infty)$

f IS DECREASING ON $(-1, 3)$

$$\therefore f(-1) = \frac{6}{3}(-1)^3 - 6(-1)^2 - 18(-1) - 6 = 4 \text{ IS A RELATIVE}$$

MAXIMUM.

$$f(3) = \frac{6}{3}(3)^3 - 6(3)^2 - 18(3) - 6 = -60 \text{ IS A RELATIVE}$$

MINIMUM.