

Last Name: SOLUTION

First Name: \_\_\_\_\_

Student ID: \_\_\_\_\_

## Quiz 2(B)

Question 1. (5 marks) Find the first and second derivative of the function:

$$f(u) = \frac{u^2+3}{u-1} \quad f'(u) = \frac{2u(u-1) - (u^2+3)(1)}{(u-1)^2} = \frac{2u^2 - 2u - u^2 - 3}{(u-1)^2}$$

$$= \frac{u^2 - 2u - 3}{(u-1)^2}$$

$$f''(u) = \frac{(2u-2)(u-1)^2 - (u^2-2u-3)[2(u-1)(1)]}{(u-1)^4}$$

$$= \frac{(u-1)[(2u-2)(u-1) - 2(u^2-2u-3)]}{(u-1)^4} = \frac{2u^2 - 2u - 2u + 2 - 2u^2 + 4u + 6}{(u-1)^3}$$

$$= \frac{8}{(u-1)^3}$$

Question 2. (5 marks) Find  $\frac{dy}{dx}$ :

$$\sqrt{xy} = y^2 + 3x$$

$$\frac{d}{dx}(xy)^{1/2} = \frac{d}{dx}(y^2) + \frac{d}{dx}(3x)$$

$$\frac{1}{2}(xy)^{-1/2}(y + xy') = 2yy' + 3$$

$$\frac{y + xy'}{2(xy)^{1/2}} = 2yy' + 3$$

$$y + xy' = 4x^{1/2}y^{3/2}y' + 6x^{1/2}y^{1/2}$$

$$xy' - 4x^{1/2}y^{3/2}y' = 6x^{1/2}y^{1/2} - y$$

$$y'(x - 4x^{1/2}y^{3/2}) = 6x^{1/2}y^{1/2} - y$$

$$y' = \frac{6x^{1/2}y^{1/2} - y}{x - 4x^{1/2}y^{3/2}}$$

**Question 3.** (5 marks) Find the intervals where the function is increasing, the intervals where the function is decreasing and any relative extrema:

$$f(x) = \frac{4}{3}x^3 - 4x^2 - 12x - 4$$

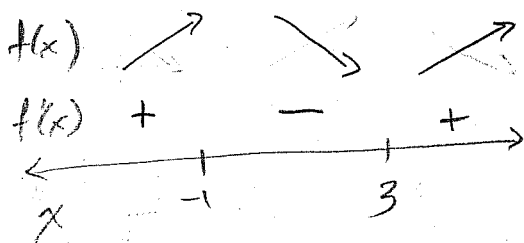
$$f'(x) = 4x^2 - 8x - 12$$

$$= 4(x^2 - 2x - 3)$$

$$= 4(x-3)(x+1)$$

$$f'(x) = 0 \Leftrightarrow x = -1, 3$$

$f(x)$  EXIST EVERYWHERE.



TEST POINTS:

$$x = -2 : f'(-2) = 4(-2)^2 - 8(-2) - 12 = 20 > 0$$

$$x = 0 : f'(0) = -12 < 0$$

$$x = 4 : f'(4) = 4(4)^2 - 8(4) - 12 = 20 > 0$$

$f$  IS INCREASING ON  $(-\infty, -1)$  AND  $(3, \infty)$ .

$f$  IS DECREASING ON  $(-1, 3)$ .

$$f(-1) = \frac{4}{3}(-1)^3 - 4(-1)^2 - 12(-1) - 4 = \frac{8}{3} \text{ IS}$$

A RELATIVE MAXIMUM

$$f(3) = \frac{4}{3}(3)^3 - 4(3)^2 - 12(3) - 4 = -40 \text{ IS}$$

A RELATIVE MINIMUM