Dawson College: Calculus I: 201-NYA-05 15

October 1, 2010

Last Name: SOLUTIONS

First Name: ____________

Student ID: ____________

Test 1

The length of the test is 1hr and 45min. You may use a nonprogrammable, nongraphing scientific calculator. Remember to clearly show all of your work and to use correct notation.

Question 1. *(4 marks)*
Simplify the following expressing your final answer with positive exponents only:

$$\left(\frac{4x^{-1}}{y}\right)\cdot\left(\frac{x^4 z^{-4}}{16x^8 z^8 y^{-3}}\right)^{-1/4} = \frac{4}{xy}\cdot\frac{x^{-1}z^{1}}{(16)^{-1/4}x^{-2}z^{-2}y^{3/4}}$$

$$= \frac{4}{xy}\cdot\frac{16^{1/4}x^2z^2z^1}{x\,y^{3/4}} = \frac{4\cdot 2x^2z^3}{x\,x\,y\,y^{3/4}}$$

$$= \frac{8x^2z^3}{x^2\,y^{7/4}} = \frac{8z^3}{y^{7/4}}$$

Question 2. *(4 marks)* Simplify the following expressing your final answer as a single rational function with positive exponents only:

$$\frac{(x-1)(2x+1)^{-1/2}+(2x+1)^{1/2}}{x^3} = \frac{\frac{x-1}{(2x+1)^{1/2}}+\frac{(2x+1)^{1/2}(2x+1)^{1/2}}{(2x+1)^{1/2}}}{x^3}$$

$$= \frac{\frac{(x-1)+(2x+1)}{(2x+1)^{1/2}}}{x^3} = \frac{3x}{(2x+1)^{1/2}}\cdot\frac{1}{x^3}$$

$$= \frac{3}{x^2(2x+1)^{1/2}}$$

Question 3. *(5 marks)* A company has determined that for a certain product the supply equation is $2p = 2x + 56$ and the demand equations is $p = -2x^2 - 2x + 55$ where x represents the quantity demanded in units of a thousand and p is the unit price in dollars. Find the equilibrium price and equilibrium quantity for the product.

$2p = 2x + 56 \Rightarrow p = x + 28$

INTERSECTION:

$x + 28 = -2x^2 - 2x + 55$

$2x^2 + 3x - 27 = 0$

$2x^2 + 9x - 6x - 27 = 0$

$x(2x+9) - 3(2x+9) = 0$

$(x-3)(2x+9) = 0$

$\therefore x = 3$ or $\cancel{x = -\frac{9}{2}}$

$\alpha \cdot \beta = -54$

$\alpha + \beta = 3$

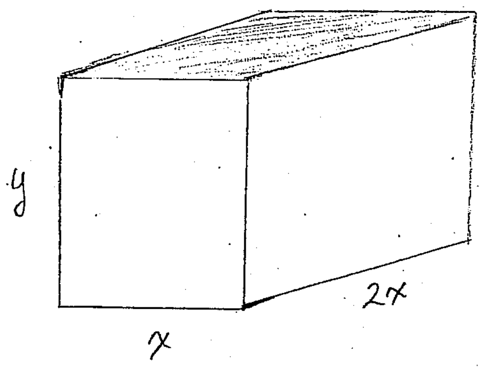
$\alpha = 9 \quad \beta = -6$

$\therefore$ THE EQUILIBRIUM QUANTITY IS 3000.

$P = 3 + 28 = 31$

$\therefore$ THE EQUILIBRIUM PRICE IS \$31.

Question 4. *(5 marks)* A rectangular storage container with an open top is to have a volume of 10m^3. The length of its base is twice the width. Material for the base costs \$10 per square metre. Material for the sides cose \$6 per square metre. Express the cost of the container as a function of x. What is the domain of this function?



$$\text{COST} = 2x^2(10) + 2xy(6) + 2(2x)(y)(6)$$
$$= 20x^2 + 12xy + 24xy$$
$$= 20x^2 + 36xy$$

$$V = (2x)(x)(y) = 10$$
$$2x^2y = 10$$
$$y = \frac{10}{2x^2} = \frac{5}{x^2}$$

$$\therefore \quad C(x) = 20x^2 + 36x\left(\frac{5}{x^2}\right)$$
$$C(x) = 20x^2 + \frac{180}{x}$$

THE ONLY RESTRICTION ON x IS THAT $x > 0$
($x \neq 0$ SINCE $V = 10\text{m}^3$)

$\therefore$ THE DOMAIN OF $C(x)$ IS $(0, \infty)$

Question 5. *(10 marks)* Evaluate the following limits:

(a) $\lim\limits_{x\to 4} \dfrac{x^2+3x-28}{x^2-16}$ $\underset{=}{\text{I.F.}\ \frac{0}{0}}$ $\lim\limits_{x\to 4} \dfrac{\cancel{(x-4)}(x+7)}{\cancel{(x-4)}(x+4)}$

$= \lim\limits_{x\to 4} \dfrac{x+7}{x+4} = \dfrac{4+7}{4+4} = \dfrac{11}{8}$

(b) $\lim\limits_{x\to 5} \dfrac{x^2-x-20}{x^2+2x+5}$ $=$ $\dfrac{(5)^2-5-20}{(5)^2+2(5)+5} = \dfrac{0}{40} = 0$

(c) $\lim\limits_{x\to 16} \dfrac{x-16}{\sqrt{x}-4} = \lim\limits_{x\to 16} \dfrac{x-16}{\sqrt{x}-4} \cdot \dfrac{\sqrt{x}+4}{\sqrt{x}+4}$

$= \lim\limits_{x\to 16} \dfrac{(x-16)(\sqrt{x}+4)}{x+4\sqrt{x}-4\sqrt{x}-16} = \lim\limits_{x\to 16} \dfrac{\cancel{(x-16)}(\sqrt{x}+4)}{\cancel{x-16}}$

$= \lim\limits_{x\to 16} \sqrt{x}+4 = \sqrt{16}+4 = 8.$

(d) $\lim_{x \to -\infty} \frac{4x^2 - 3x + 5}{x^3 - 4x^2 + 2} = \lim_{x \to -\infty} \dfrac{\frac{4x^2}{x^3} - \frac{3x}{x^3} + \frac{5}{x^3}}{\frac{x^3}{x^3} - \frac{4x^2}{x^3} + \frac{2}{x^3}}$

$$= \lim_{x \to -\infty} \frac{\frac{4}{x} - \frac{3}{x^2} + \frac{5}{x^3}}{1 - \frac{4}{x} + \frac{2}{x^3}} = \frac{0 - 0 + 0}{1 - 0 + 0} = 0$$

(e) $\lim_{x \to \infty} \frac{-2x^3 + 5x - 1}{4x^3 - 5x + 7}$

$$= \lim_{x \to -\infty} \frac{\frac{-2x^3}{x^3} + \frac{5x}{x^3} - \frac{1}{x^3}}{\frac{4x^3}{x^3} - \frac{5x}{x^3} + \frac{7}{x^3}}$$

$$= \lim_{x \to -\infty} \frac{-2 + \frac{5}{x^2} - \frac{1}{x^3}}{4 - \frac{5}{x^2} + \frac{7}{x^3}}$$

$$= \frac{-2 + 0 - 0}{4 - 0 + 0}$$

$$= \frac{-1}{2}$$

Question 6. *(5 marks)* Find the values of x for which the following funtion is continuous (explain your reasoning):

$$f(x) = \begin{cases} 3x+1 & \text{if } x<3 \\ 10 & \text{if } x=3 \\ x^2-x+4 & \text{if } x>3 \end{cases}$$

$f(x)$ IS CONTINUOUS ON $(-\infty, 3) \cup (3, \infty)$ SINCE IT IS A POLYNOMIAL THERE.

CHECK THE THREE CONDITIONS FOR CONTINUITY AT $x=3$.

1) $f(3) = 10$ SO $f(3)$ IS DEFINED.

2) $\lim_{x \to 3^-} f(x) = \lim_{x \to 3^-} 3x+1 = 3(3)+1 = 10$

$\lim_{x \to 3^+} f(x) = \lim_{x \to 3^+} x^2 - x + 4 = (3)^2 - 3 + 4 = 10$

$\therefore \lim_{x \to 3^-} f(x) = \lim_{x \to 3^+} f(x) = 10$ SO $\lim_{x \to 3} f(x)$ EXISTS (AND IS 10)

3) $\lim_{x \to 3} f(x) = 10 = f(3)$

$\therefore f$ IS CONTINUOUS AT $x=3$.

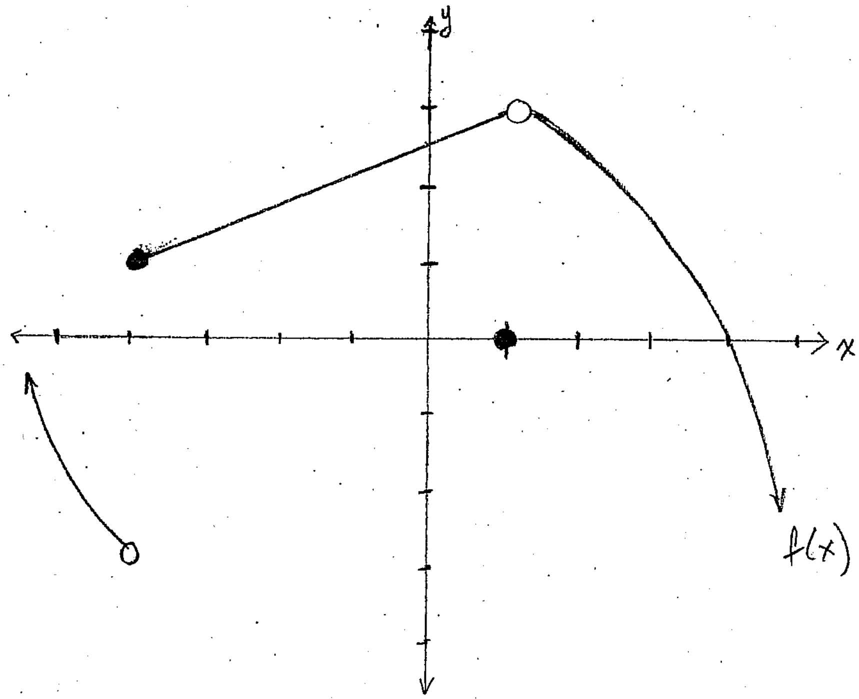
$\therefore f$ IS CONTINUOUS ON $(-\infty, \infty)$

Question 7. *(6 marks)* Given the following graph, find the following if it exists:

(a) $\lim_{x \to -4^-} f(x), \; \lim_{x \to -4^+} f(x), \; \lim_{x \to -4} f(x)$

(b) $\lim_{x \to 1} f(x), \; f(1)$

(c) Is $f(x)$ continuous at $x = 1$? Why or why not?



a) $\lim_{x \to -4^-} f(x) = -3$, $\lim_{x \to -4^+} f(x) = 1$, $\lim_{x \to -4} f(x)$ DOES NOT EXIST

b) $\lim_{x \to 1} f(x) = 3$, $f(1) = 0$

c) $f(x)$ IS NOT CONTINUOUS AT $x = 1$ SINCE

$\lim_{x \to 1} f(x) \neq f(1)$ (FAILS CONDITION 3).

Bonus Question. *(3 marks)* Find the values of a and b that make the following function continuous on all real numbers.

$$f(x) = \begin{cases} 2ax+b & \text{if} \quad x<1 \\ 2bx^2+4 & \text{if} \quad 1 \le x \le 2 \\ ax-14 & \text{if} \quad x>2 \end{cases}$$

f IS CONTINUOUS EVERYWHERE EXCEPT POSSIBLY AT $x=1$ AND $x=2$

AT $x=1$

1) $f(1) = 2b(1)^2+4 = 2b+4$

2) $\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} 2ax+b = 2a(1)+b = 2a+b$

$\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} 2bx^2+4 = 2b(1)^2+4 = 2b+4$

$\therefore$ WE NEED $2a+b = 2b+4$ FOR $\lim_{x \to 1} f(x)$ TO EXIST.

3) AGAIN WE WANT $\underbrace{2a+b = 2b+4}_{\lim_{x \to 1} f(x)} = \underbrace{2b+4}_{f(1)}$

$\therefore$ IF $2a+b = 2b+4$ f IS CONTINUOUS AT $x=1$

$\Rightarrow$ $2a-b=4$

AT $x=2$

1) $f(2) = 2b(2)^2+4 = 8b+4$

2) $\lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} 2bx^2+4 = 2b(2)^2+4 = 8b+4$

$\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} ax-14 = a(2)-14 = 2a-14$

$\therefore$ WE NEED $2a-14 = 8b+4$ FOR $\lim_{x \to 2} f(x)$ TO EXIST

3) AGAIN WE NEED $\underbrace{2a-14 = 8b+4}_{\lim_{x \to 2} f(x)} = \underbrace{8b+4}_{f(1)}$

$\therefore$ IF $2a-14 = 8b+4$ f IS CONTINUOUS AT $x=2$

$\Rightarrow$ $8b-2a=-18$

NOW $2a - b = 4 \Rightarrow 2a = b + 4$

$8b - 2a = -18 \Rightarrow 8b - (b+4) = -18)$

$7b - 4 = -18$

$7b = -14$

$b = -2$

$\therefore 2a = b + 4$

$= -2 + 4$

$= 2$

$a = 1$

$\therefore$ IF $a = 1$ AND $b = -2$ f IS CONTINUOUS ON ALL REAL NUMBERS.