Dawson College: Calculus I: 201-NYA-05 15

October 29, 2010

Last Name: SOLUTIONS

First Name: ____________

Student ID: ____________

Test 2

The length of the test is 1hr and 45min. You may use a nonprogrammable, nongraphing scientific calculator. Remember to clearly show all of your work and to use **correct notation**.

Question 1. *(5 marks)* Use the (limit) definition of the dervative to find the derivative of $f(x) = 2x^2 - x$.

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{[2(x+h)^2 - (x+h)] - [2x^2 - x]}{h}$$

$$= \lim_{h\to 0} \frac{2(x^2+2xh+h^2) - x - h - 2x^2 + x}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{2x^2} + 4xh + 2h^2 - \cancel{x} - h - \cancel{2x^2} + \cancel{x}}{h}$$

$$= \lim_{h\to 0} \frac{4xh + 2h^2 - h}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{h}(4x + 2h - 1)}{\cancel{h}}$$

$$= \lim_{h\to 0} (4x + 2h - 1)$$

$$= 4x + 0 - 1$$

$$= 4x - 1$$

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Question 2. (~~16~~ *marks)* Find the derivatives of the following functions. **Do not simplify your answer**.

(a) $f(x) = \frac{3}{x^3} + \frac{2}{\sqrt{x}} - \sqrt[3]{x^2}$ $= 3x^{-3} + 2x^{-1/2} - x^{2/3}$

$f'(x) = -9x^{-4} - x^{-3/2} - \frac{2}{3}x^{-1/3}$

$= -\frac{9}{x^4} - \frac{1}{x^{3/2}} - \frac{2}{3x^{1/3}}$

(b) $g(x) = \left(\frac{3x+1}{2x-2}\right)^{3/2}$

$g'(x) = \frac{3}{2}\left(\frac{3x+1}{2x-2}\right)^{1/2} \cdot \frac{d}{dx}\left(\frac{3x+1}{2x-2}\right)$

$= \frac{3}{2}\left(\frac{3x+1}{2x-2}\right)^{1/2} \frac{3(2x-2)-(2)(3x+1)}{(2x-2)^2}$

(c) $F(x) = (4x^3 - 3x^2 + 5)\left(x^2 + 2x - \frac{3}{x}\right)$

$$F'(x) = \frac{d}{dx}(4x^3-3x^2+5)\cdot\left(x^2+2x-\frac{3}{x}\right) + (4x^3-3x^2+5)\frac{d}{dx}\left(x^2+2x-\frac{3}{x}\right)$$

$$= (12x^2-6x+0)\left(x^2+2x-\frac{3}{x}\right) + (4x^3-3x^2+5)\left(2x+2+\frac{3}{x^2}\right)$$

Question 3. *(3 marks)* Find all x values where the tangent line to

$$f(x) = x^3 + \frac{11}{2}x^2 - 4x + 7$$

is horizontal.

$$f'(x) = 3x^2 + 11x - 4 = 0$$

$$3x^2 + 12x - x - 4 = 0$$

$$3x(x+4) - (x+4) = 0$$

$$(3x-1)(x+4) = 0$$

$$x = -4, \tfrac{1}{3}$$

Question 4. *(3 marks)* Let

$$h(x) = \frac{g(x) - x}{f(x)}$$

Given $f(-1) = 3$, $g(-1) = -2$, $f'(-1) = 2$, and $g'(-1) = 4$, find $h'(-1)$

$$h'(x) = \frac{\frac{d}{dx}[g(x) - x]\,f(x) - [g(x) - x]\frac{d}{dx}[f(x)]}{[f(x)]^2}$$

$$= \frac{[g'(x) - 1]f(x) - [g(x) - x]f'(x)}{[f(x)]^2}$$

$$h'(-1) = \frac{[g'(-1) - 1]f(-1) - [g(-1) - (-1)]f'(-1)}{[f(-1)]^2}$$

$$= \frac{[4-1](3) - [-2+1](2)}{(3)^2}$$

$$= \frac{11}{9}$$

Question 5. **(a)** *(4 marks)* The following is a problem from a calculus test that I took in University and my solution which is incorrect. Find and expain the four main mistakes that I made (only four, not bad!). (Note: full marks will not be given for only finding the mistakes without sufficient explanation). You will be asked to solve the problem in part **(b)**.

Problem: Find an equation of the tangent line to the graph of $f(x) = (2x-1)^2(x^2-x+2)$ at $(1,2)$.

"Solution":

$$\begin{aligned} f'(x) &= \frac{d}{dx}\left[(2x-1)^2\right] \cdot \frac{d}{dx}(x^2-x+2) \\ &= 2(2x-1)(2)\cdot\underline{(2x-0)} \\ &= 4(2x-1)(2x) \end{aligned}$$

← NEED TO USE PRODUCT RULE

NOT THE DERIVATIVE OF x^2-x+2 SINCE $\frac{d}{dx}(x) = 1$ NOT 0.

$$\begin{aligned} f'(2) &= 4\left[2(2)-1\right]\left[2(2)\right] \\ &= 4(3)(4) \\ &= 48 \end{aligned}$$

USED y INSTEAD OF $x = 1$

So the slope of the tangent line is $m = 48$

Tangent line:

$$\begin{aligned} y &= mx+b \\ 1 &= 48(2)+b \\ -95 &= b \end{aligned}$$

← SWITCHED x AND y

Therefore the equation of the tangent line is:

$y = 48x - 95$

(b) *(5 marks)* Write a correct solution for the problem in part **(a)**.

$f(x) = (2x-1)^2 (x^2 - x + 2)$

$f'(x) = \frac{d}{dx}\left[(2x-1)^2\right](x^2 - x + 2) + (2x-1)^2 \frac{d}{dx}(x^2 - x + 2)$

$= 2(2x-1)\cdot(2)(x^2 - x + 2) + (2x-1)^2(2x-1)$

$f'(1) = 4(2\cdot1 - 1)(1^2 - 1 + 2) + (2\cdot1 - 1)^2(2\cdot1 - 1)$

$= 4(1)(2) + (1)^2(1)$

$= 9$

$\therefore$ THE SLOPE OF THE TANGENT LINE IS $m = 9$

$y = mx + b$

$2 = 9(1) + b$

$-7 = b$

$\therefore \boxed{y = 9x - 7}$

Question 6. *(6 marks)* The weekly demand function for a certain product is

$$p = -0.07x + 800$$

where p denotes the wholesale price in dollars and x denotes the quantity demanded. The weekly total cost function asociated witht manufacturing the product is given by

$$C(x) = 0.0000003x^3 - 0.03x^2 + 400x + 80000$$

where $c(x)$ denotes the total cost incurred in producing x units of the product.

(a) Find the revenue function $R(x)$ and the profit funtion $P(x)$.

$$R(x) = x \cdot p = x(-0.07x + 800) = -0.07x^2 + 800x$$

$$P(x) = R(x) - C(x)$$

$$= -0.07x^2 + 800x - (0.0000003x^3 - 0.03x^2 + 400x + 80000)$$

$$= -0.0000003x^3 - 0.04x^2 + 400x - 80000$$

(b) Find the marginal cost function and the marginal profit function. (only!)

$$C'(x) = 0.0000009x^2 - 0.06x + 400$$

$$P'(x) = -0.0000009x^2 - 0.08x + 400$$

(c) Compute $C'(1800)$. What does this value tell us?

$$C'(1800) = 0.0000009(1800)^2 - 0.06(1800) + 400$$

$$= 294.92.$$

THIS TELLS US THAT THE COST OF PRODUCING THE 1801th UNIT IS APPROXIMATELY \$294.92

Question 7. *(5 marks)* The weekly demand function for a certain product is

$$p = -0.04x + 600$$

where p denotes the wholesale price in dollars and x denotes the quantity demanded.

(a) Find the elasticity of demand $E(p)$.

$$0.04x = -p + 600$$

$$x = -25p + 15000 = f(p)$$

$$f'(p) = -25$$

$$E(p) = -\frac{p\,f'(p)}{f(p)} = -\frac{p(-25)}{-25p + 15000} = \frac{25p}{15000 - 25p}$$

$$= \frac{25p}{25(600 - p)} = \frac{p}{600 - p}$$

(b) Is demand elastic, inelastic or unitary when price is \$400? What happens to revenue if the price is raised slightly from \$400?

$$E(400) = \frac{400}{600-400} = 2 > 1$$

$\therefore$ DEMAND IS ELASTIC AT \$400.

$\therefore$ IF THE PRICE IS RAISED SLIGHTLY FROM \$400 REVENUE WILL DECREASE.