

Bonus Assignment 1

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

§7.1 #8

Intersection:

$$1 + \sqrt{x} = 1 + \frac{1}{3}x$$

$$\sqrt{x} = \frac{1}{3}x$$

$$(\sqrt{x})^2 = \left(\frac{1}{3}x\right)^2$$

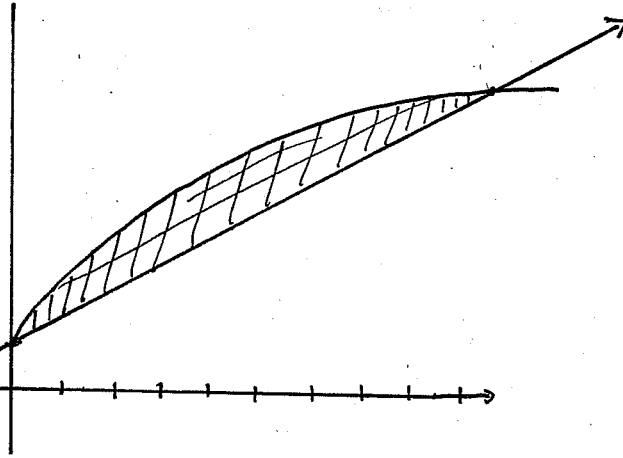
$$x = \frac{1}{9}x^2$$

$$0 = \frac{1}{9}x^2 - x$$

$$0 = x\left(\frac{1}{9}x - 1\right)$$

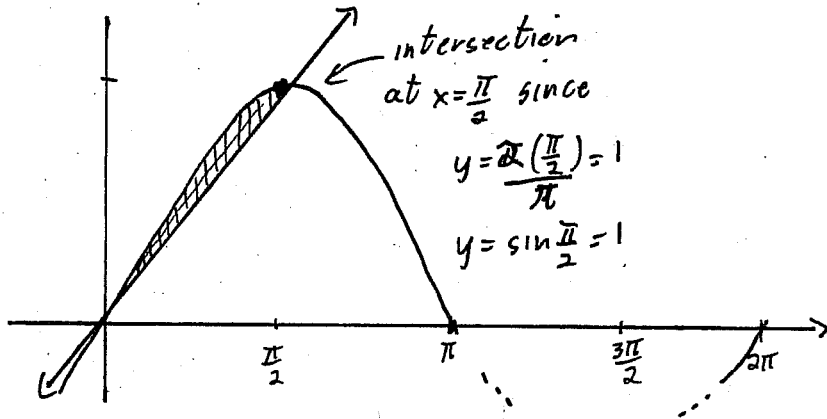
$$x = 0$$

$$\begin{aligned} \frac{1}{9}x - 1 &= 0 \\ \frac{1}{9}x &= 1 \\ x &= 9 \end{aligned}$$



$$\begin{aligned} A &= \int_0^9 \left(1 + \sqrt{x} - \left(1 + \frac{1}{3}x\right)\right) dx \\ &= \int_0^9 \left(\sqrt{x} - \frac{1}{3}x\right) dx \\ &= \left[\frac{2x^{3/2}}{3} - \frac{x^2}{6} \right]_0^9 \\ &= \left[\frac{2(9)^{3/2}}{3} - \frac{9^2}{6} \right] = 18 - \frac{27}{2} \\ &= \frac{9}{2} \end{aligned}$$

§7.1 #14



intersection at $x = \frac{\pi}{2}$ since
 $y = \frac{2}{\pi} \left(\frac{\pi}{2}\right) = 1$
 $y = \sin \frac{\pi}{2} = 1$

$$\begin{aligned} A &= \int_0^{\pi/2} \left(\sin x - \frac{2x}{\pi}\right) dx \\ &= \left[-\cos x - \frac{x}{\pi} \right]_0^{\pi/2} \\ &= \left[-\cos \frac{\pi}{2} - \frac{1}{\pi} \left(\frac{\pi}{2}\right)^2 \right] - \left[-\cos 0 - \frac{1}{\pi} 0^2 \right] \\ &= 1 - \frac{\pi}{4} \end{aligned}$$

§7.1 #12

$$\begin{aligned} 4x + y^2 &= 12 \\ 4x &= -y^2 + 12 \\ x &= \frac{-y^2 + 12}{4} \end{aligned}$$

X-int: (3, 0)

Intersection:

$$y = \frac{-1}{4}y^2 + 3$$

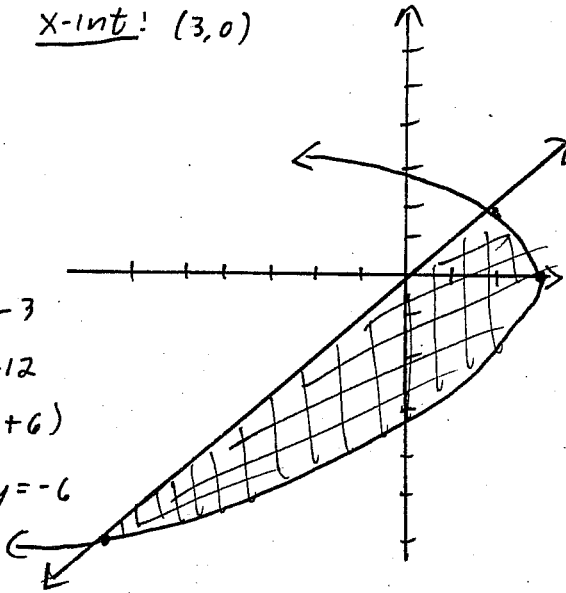
$$0 = \frac{1}{4}y^2 + y - 3$$

$$0 = y^2 + 4y - 12$$

$$0 = (y-2)(y+6)$$

$$y = 2$$

$$y = -6$$

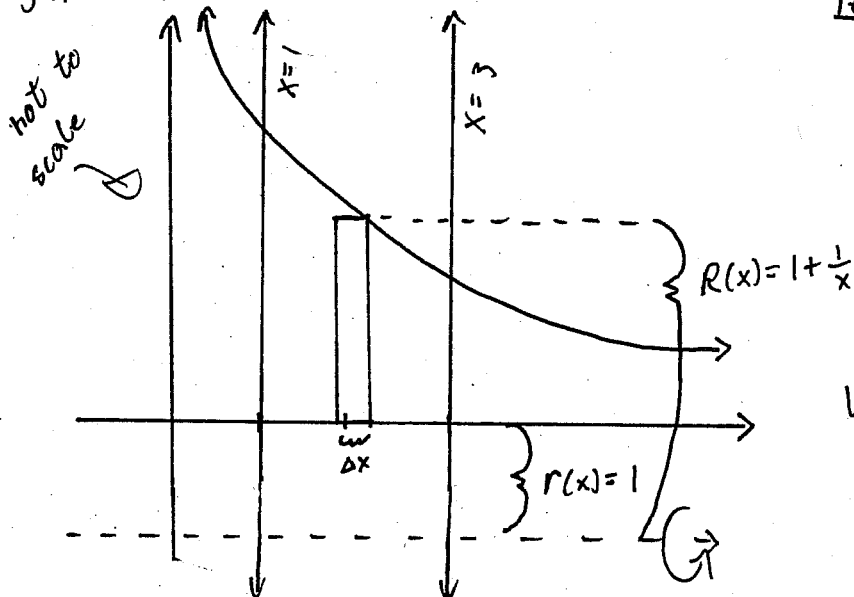


$$\begin{aligned} A &= \int_{-6}^2 \left(\frac{-1}{4}y^2 + 3 - y\right) dy \\ &= \left[\frac{-y^3}{12} + 3y - \frac{y^2}{2} \right]_{-6}^2 \\ &= \left[\frac{-2^3}{12} + 3(2) - \frac{2^2}{2} \right] - \left[\frac{(-6)^3}{12} + 3(-6) - \frac{(-6)^2}{2} \right] \\ &= \left[\frac{-8}{12} + 6 - 2 \right] - \left[18 - 18 - 18 \right] \\ &= -\frac{2}{3} + 4 + 18 = \frac{64}{3} \end{aligned}$$

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§ 7.2 #10

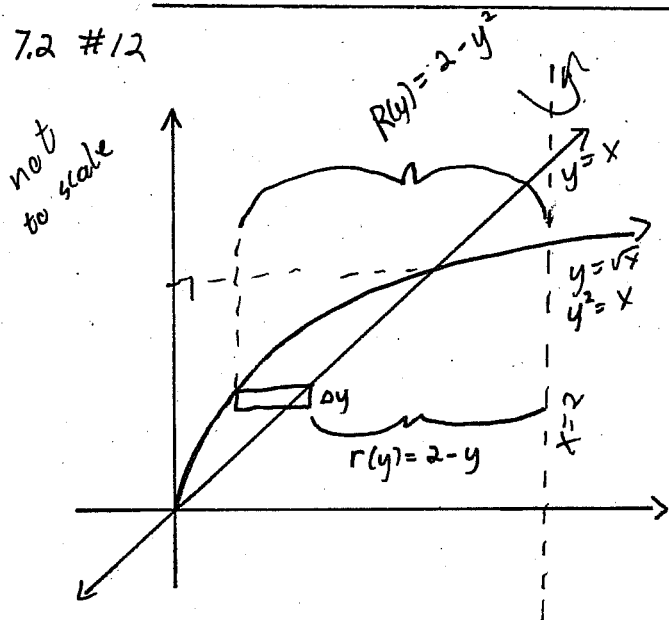


rep. element:

$$\begin{aligned} \Delta V &= \pi \left[(R(x))^2 - (r(x))^2 \right] \Delta x \\ &= \pi \left[\left(1 + \frac{1}{x}\right)^2 - 1^2 \right] \Delta x \\ &= \pi \left[1 + \frac{2}{x} + \frac{1}{x^2} - 1 \right] \Delta x \\ &= \pi \left[\frac{1}{x^2} + \frac{2}{x} \right] \Delta x \end{aligned}$$

$$\begin{aligned} V &= \int_1^3 \pi \left[\frac{1}{x^2} + \frac{2}{x} \right] dx \\ &= \pi \left[-\frac{1}{x} + 2 \ln|x| \right]_1^3 \\ &= \pi \left[-\frac{1}{3} + 2 \ln 3 \right] - \pi \left[-\frac{1}{1} + 2 \ln 1 \right] \\ &= \pi - \frac{1}{3}\pi + 2\pi \ln 3 = \frac{2}{3}\pi + 2\pi \ln 3 \end{aligned}$$

§ 7.2 #12



rep. element:

$$\begin{aligned} \Delta V &= \pi \left[(R(y))^2 - (r(y))^2 \right] \Delta y \\ &= \pi \left[(2 - y^2)^2 - (2 - y)^2 \right] \Delta y \\ &= \pi \left[4 - 4y^2 + y^4 - 4 + 4y - y^2 \right] \Delta y \\ &= \pi \left[y^4 - 5y^2 + 4y \right] \Delta y \end{aligned}$$

$$\begin{aligned} V &= \int_0^1 \pi \left[y^4 - 5y^2 + 4y \right] dy \\ &= \pi \left[\frac{y^5}{5} - \frac{5y^3}{3} + 2y^2 \right]_0^1 \\ &= \pi \left[\frac{1}{5} - \frac{5}{3} + 2 \right] \\ &= \pi \left[\frac{3}{15} - \frac{15}{15} + \frac{30}{15} \right] = \frac{18}{15}\pi = \frac{6}{5}\pi \end{aligned}$$