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Bonus Quiz 2

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

① $\sum_{n=1}^{\infty} \ln\left(\frac{n}{3n+1}\right)$ Let $a_n = \ln\left(\frac{n}{3n+1}\right)$

$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \ln\left(\frac{n}{3n+1}\right) = \ln\left(\frac{1}{3}\right) \neq 0$

$\therefore$ n^{th} term divergence test the series diverges

② $\sum_{n=2}^{\infty} \frac{1}{n\sqrt{\ln n}}$ Let $f(x) = \frac{1}{x\sqrt{\ln x}}$

- $f(x)$ positive for $x \in [2,\infty)$? ✓
- $f(x)$ continuous for $x \in [2,\infty)$? ✓
- $f(x)$ decreasing for $x \in [2,\infty)$?

$$f'(x) = \frac{-1\left(\sqrt{\ln x} + x\frac{1}{2}(\ln x)^{-\frac{1}{2}}\right)}{\left(x\sqrt{\ln x}\right)^2} < 0 \quad ✓$$

$$\int_2^{\infty} \frac{1}{x\sqrt{\ln x}}\,dx = \lim_{b\to\infty} \int_2^{b} \frac{1}{x\sqrt{\ln x}}\,dx$$

$u = \ln x$, $du = \frac{1}{x}dx$, $u(2) = \ln 2$, $u(b) = \ln b$

$$= \lim_{b\to\infty} \int_{\ln 2}^{\ln b} \frac{1}{\sqrt{u}}\,du$$

$$= \lim_{b\to\infty} \left[2\sqrt{u}\right]_{\ln 2}^{\ln b}$$

$$= \lim_{b\to\infty} \left[2\sqrt{\ln b} - 2\sqrt{\ln 2}\right]$$

($\sqrt{\ln b} \to \infty$)

diverges

$\therefore$ by integral test the series diverges.