

## Bonus Quiz 2

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

①  $\sum_{n=1}^{\infty} \frac{2n}{n+1}$       Let  $a_n = \frac{2n}{n+1}$

$\lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{2n}{n+1} = 2 \neq 0$        $\therefore$   $n^{\text{th}}$  term divergence test  
 the series diverges.

②  $\sum_{n=1}^{\infty} \frac{\ln n}{n^2}$       Let  $f(x) = \frac{\ln x}{x^2}$

- $f(x)$  positive for  $x \in [1, \infty)$ ? ✓
- $f(x)$  continuous for  $x \in [1, \infty)$ ? ✓
- $f(x)$  decreasing for  $x \in [1, \infty)$ ?  $f'(x) = \frac{\frac{1}{x}x^2 - 2x \ln x}{x^4} = \frac{x(1 - 2 \ln x)}{x^4} < 0$  ✓

$\int_1^{\infty} \frac{\ln x}{x^2} dx = \lim_{b \rightarrow \infty} \int_1^b \frac{\ln x}{x^2} dx$        $u = \ln x$        $du = \frac{1}{x} dx$   
 $v = \frac{-1}{x}$        $dv = \frac{-1}{x^2} dx$

$= \lim_{b \rightarrow \infty} \left[ [uv]_1^b - \int_1^b v du \right]$   
 $= \lim_{b \rightarrow \infty} \left[ \left[ \frac{-\ln x}{x} \right]_1^b - \int_1^b \frac{-1}{x^2} dx \right]$   
 $= \lim_{b \rightarrow \infty} \left[ \frac{-\ln b}{b} + \frac{\ln 1}{1} - \left[ \frac{1}{x} \right]_1^b \right]$   
 $= \lim_{b \rightarrow \infty} \left[ \frac{-\ln b}{b} - \frac{1}{b} + 1 \right] = 1$        $\therefore$  by integral test the series converges.

Converges