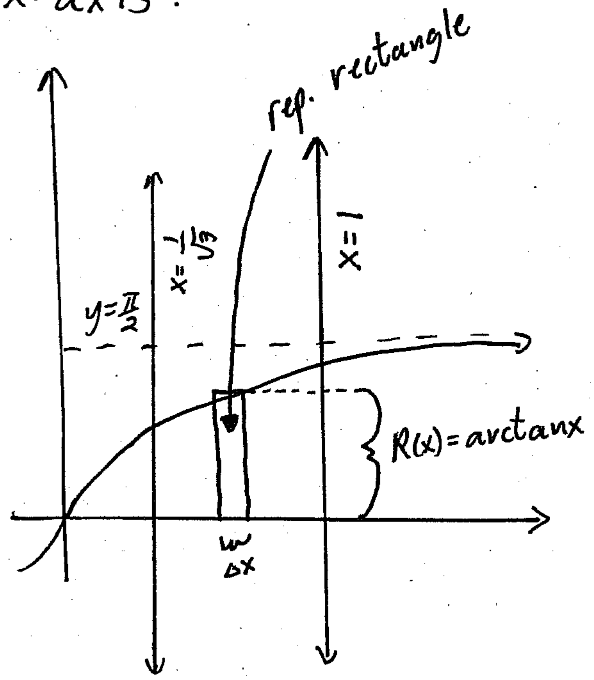
Disk Method: Example

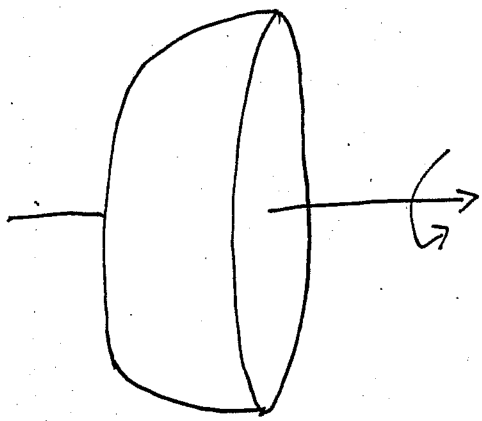
Find the volume of a solid of revolution formed by rotating the region bounded by the following: $f(x) = \arctan x$, $y=0$, $x = \frac{1}{\sqrt{3}}$, $x=1$ about the x-axis.



Rep. element:

$\Delta V = \pi (\arctan x)^2 \Delta x$

$\Delta V = \pi \arctan^2 x \, \Delta x$



$$V = \int_{1/\sqrt{3}}^{1} \pi \arctan^2 x \, dx$$

$$= \pi \int_{1/\sqrt{3}}^{1} \arctan^2 x \, dx$$

$u = (\arctan x)^2 \qquad du = \frac{2\arctan x}{x^2+1} dx$

$v = x \qquad dv = dx$

$$= \pi \left[\left[uv \right]_{1/\sqrt{3}}^{1} - \int_{1/\sqrt{3}}^{1} v \, du \right]$$

$$= \pi \left[\left[x \arctan^2 x \right]_{1/\sqrt{3}}^{1} - \int_{1/\sqrt{3}}^{1} \frac{2x \arctan x}{x^2+1} dx \right]$$

$u = \arctan x \qquad du = \frac{dx}{x^2+1}$

$v = \ln(x^2+1) \qquad dv = \frac{2x}{x^2+1} dx$

$$= \pi \left[\left[x \arctan^2 x \right]_{1/\sqrt{3}}^{1} - \left[\left[\ln(x^2+1) \arctan x \right]_{1/\sqrt{3}}^{1} - \int_{1/\sqrt{3}}^{1} \frac{2x \ln(x^2+1)}{x^2+1} dx \right] \right]$$

$u = \ln(x^2+1) \qquad u(1/\sqrt{3}) = \ln\left(\left(\frac{1}{\sqrt{3}}\right)^2+1\right) = \ln\left(\frac{1}{3}+1\right) = \ln\left(\frac{4}{3}\right)$

$du = \frac{2x}{x^2+1}\,dx \qquad u(1) = \ln(1^2+1) = \ln 2$

$$= \pi\left[\left[(\arctan 1)^2 - \frac{1}{\sqrt{3}}\left(\arctan\frac{1}{\sqrt{3}}\right)^2\right] - \left[\ln(2)\arctan 1 - \ln\left(\left(\frac{1}{\sqrt{3}}\right)^2+1\right)\arctan\frac{1}{\sqrt{3}}\right]\right.$$

$$\left. + \int_{\ln\frac{4}{3}}^{\ln 2} u\,du\right.$$

$$= \pi\left[\left(\frac{\pi}{4}\right)^2 - \frac{1}{\sqrt{3}}\left(\frac{\pi}{6}\right)^2 - (\ln 2)\frac{\pi}{4} + \left(\ln\frac{4}{3}\right)\frac{\pi}{6} + \left[\frac{u^2}{2}\right]_{\ln\frac{4}{3}}^{\ln 2}\right]$$

$$= \pi\left[\frac{\pi^2}{16} - \frac{\pi^2}{\sqrt{3}(36)} - (\ln 2)\frac{\pi}{4} + \left(\ln\left(\frac{4}{3}\right)\right)\frac{\pi}{6} + \frac{(\ln 2)^2}{2} - \frac{\left(\ln\left(\frac{4}{3}\right)\right)^2}{2}\right]$$