§6.3 # 34

$$\int \frac{x^4+1}{x(x^2+1)^2}\,dx$$

$$\frac{x^4+1}{x(x^2+1)^2} = \frac{A}{x} + \frac{Bx+C}{x^2+1} + \frac{Dx+E}{(x^2+1)^2}$$

$$\frac{(x^4+1)x\cancel{(x+1)^2}}{\cancel{x(x^2+1)^2}} = \frac{Ax(x^2+1)^2}{\cancel{x}} + \frac{(Bx+C)(x)(x^2+1)^2}{\cancel{x^2+1}} + \frac{(Bx+E)x\cancel{(x^2+1)^2}}{\cancel{(x^2+1)^2}}$$

$$x^4+1 = A(x^2+1)^2 + (Bx+C)x(x^2+1) + (Dx+E)x$$

Let $x=0$

$$1 = A$$

Let $x=1$

$$1^4+1 = A(1^2+1)^2 + (B(1)+C)1(1^2+1) + (D(1)+E)1$$

$$2 = 1(2)^2 + (B+C)(2) + D+E$$

$$-2 = 2B+2C+D+E \quad (1)$$

Let $x=-1$

$$(-1)^4+1 = A((-1)^2+1)^2 + (B(-1)+C)(-1)((-1)^2+1) + (D(-1)+E)(-1)$$

$$2 = 4 + 2B - 2C + D - E$$

$$-2 = 2B - 2C + D - E \quad (2)$$

Let $x=2$

$$2^4+1 = A((2)^2+1)^2 + (B(2)+C)(2)(2^2+1) + (D(2)+E)(2)$$

$$17 = 1(25) + 20B + 10C + 4D + 2E$$

$$-8 = 20B + 10C + 4D + 2E \quad (3)$$

Let $x=-2$

$$(-2)^4+1 = A((-2)^2+1)^2 + (B(-2)+C)(-2)((-2)^2+1) + (D(-2)+E)(-2)$$

$$17 = A(25) + 20B - 10C + 4D - 2E$$

$$-8 = 20B - 10C + 4D + 2E \quad (4)$$

(1) + (2):

$$-4 = 4B + 2D$$

$$-2 = 2B + D$$

$$D = -2-2B \quad (5)$$

(3) + (4):

$$-16 = 40B + 8D$$

$$-2 = 5B + D$$

$$D = -2-5B \quad (6)$$

(5) = (6):

$$-2-2B = -2-5B$$

$$0 = B$$

$\therefore D = -2$

Sub $B=0$ and $D=-2$ into (1), (2)

(1): $-2 = 2(0) + 2C - 2 + E$

$-2C = E \quad (7)$

(2): $-2 = 2(0) - 2C - 2 - E$

$E = +2C \quad (8)$

(7) = (8):

$$-2C = 2C$$

$$C = 0$$

$\therefore E = 0$

$$\therefore \int \frac{x^4+1}{x(x^2+1)^2}\,dx = \int \frac{1}{x} - \frac{2x}{(x^2+1)^2}\,dx$$

$$= \int \frac{1}{x}\,dx - \int \frac{2x}{(x^2+1)^2}\,dx$$

$u = x^2 + 1$

$du = 2x\,dx$

$$= \ln|x| - \int \frac{du}{u^2}$$

$$= \ln|x| + \frac{1}{u} + C$$

$$= \ln|x| + \frac{1}{x^2+1} + C$$