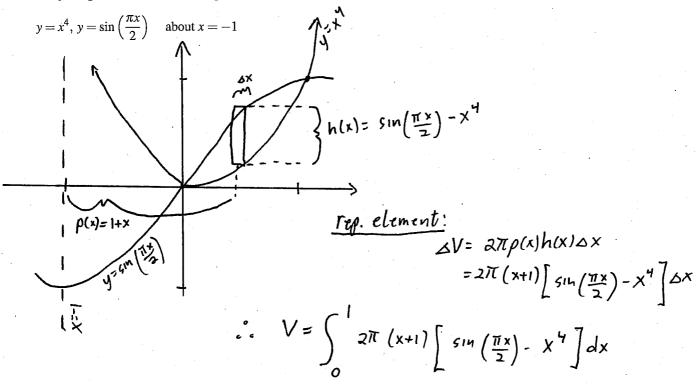Name: Y. Lamontagne

Student ID: _____

Quiz 10

This quiz is graded out of 15 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. §7.4 #23 *(5 marks)* Set up, but do not evaluate, an integral for the volume of the solid obtained by rotating the region bounded by the given curves about the specified line.

$$y = x^4, \quad y = \sin\left(\frac{\pi x}{2}\right) \quad \text{about } x = -1$$

$$h(x) = \sin\left(\frac{\pi x}{2}\right) - x^4$$

$p(x) = 1+x$

$y = \sin\left(\frac{\pi x}{2}\right)$

rep. element:

$$\Delta V = 2\pi p(x) h(x) \Delta x$$

$$= 2\pi (x+1)\left[\sin\left(\frac{\pi x}{2}\right) - x^4\right]\Delta x$$

$$\therefore \quad V = \int_0^1 2\pi (x+1)\left[\sin\left(\frac{\pi x}{2}\right) - x^4\right] dx$$

Question 2. §7.4 #9 *(5 marks)* Find the length of the curve.

$$y = \ln(\sec x), \quad 0 \le x \le \frac{\pi}{4}$$

$$y' = \frac{1}{\sec x} \sec x \tan x = \tan x$$

$$S = \int_0^{\pi/4} \sqrt{1 + (y')^2}\, dx$$

$$= \int_0^{\pi/4} \sqrt{1 + \tan^2 x}\, dx$$

$$= \int_0^{\pi/4} \sqrt{\sec^2 x}\, dx$$

$$= \int_0^{\pi/4} \sec x\, dx$$

$$= \Big[\ln|\sec x + \tan x|\Big]_0^{\pi/4}$$

$$= \ln\left|\sec\frac{\pi}{4} + \tan\frac{\pi}{4}\right| - \ln|\sec 0 + \tan 0|$$

$$= \ln|\sqrt{2} + 1| - \ln|1 + 0|$$

$$= \ln|\sqrt{2} + 1|$$

Question 3. *(5 marks)* Evaluate the indefinite integral:

$$\int \frac{t^2 - 3t - 5}{t^3 + 5t} \, dt$$

see test #2