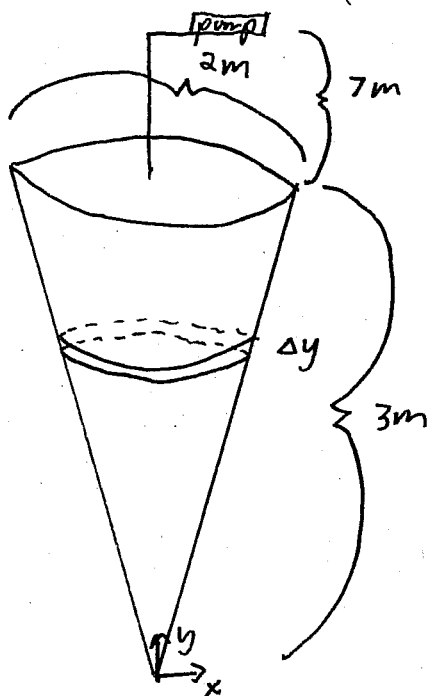


Quiz 11

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. §7.4 #24 (5 marks) A conic tank is filled with liquid chocolate which has a density of $\rho = 1200 \frac{\text{kg}}{\text{m}^3}$. If the tank is 2m across the top and has a height of 3m, set up the integral that represent the work performed to empty half the tank of chocolate through a pipe that extends 7m above the top edge? ($g = 9.8 \frac{\text{m}}{\text{s}^2}$)



Volume of slice: $\Delta V = \pi r^2 \Delta y$
 $= \pi x^2 \Delta y$
 $= \pi \left(\frac{y}{3}\right)^2 \Delta y$
 $= \frac{\pi y^2}{9} \Delta y$

mass of slice: $\Delta m = \rho \Delta V$
 $= 1200 \frac{\pi y^2}{9} \Delta y$
 $= \frac{400}{3} \pi y^2 \Delta y$

force of slice: $\Delta F = \Delta m g$
 $= \frac{400}{3} \pi y^2 \cdot 9.8 \Delta y$
 $= \frac{3920}{3} \pi y^2 \Delta y$

distance of slice to pump: $d = 10 - y$

work to move slice: $\Delta W = \Delta F d$
 $= \frac{3920}{3} \pi y^2 (10 - y) \Delta y$

work: $W = \int_{3/2}^3 \frac{3920}{3} \pi y^2 (10 - y) dy$

Question 2. §8.1 #7 (2 marks) Find a formula for the general term a_n of the sequence, assuming that the pattern of the first few terms continues.

$$\{2, 7, 12, 17, \dots\}$$

$$a_1 = 2$$

$$a_2 = 7$$

$$a_3 = 12$$

$$a_4 = 17$$

⋮

$$a_n = 5(n-1) + 2 \\ = 5n - 3$$

Question 3. §8.1 #17 (3 marks) Determine whether the sequence converges or diverges. If it converges, find the limit.

$$\left\{ \frac{e^n + e^{-n}}{e^{2n} - 1} \right\}$$

$$a_n = \frac{e^n + e^{-n}}{e^{2n} - 1}$$

$$\text{Let } f(x) = \frac{e^x + e^{-x}}{e^{2x} - 1}$$

$$\text{then } \lim_{x \rightarrow \infty} \frac{e^x + e^{-x}}{e^{2x} - 1} \quad \text{i.f. } \frac{\infty}{\infty}$$

$$= \lim_{x \rightarrow \infty} \frac{e^x - e^{-x}}{2e^{2x}} \quad \text{by } \hat{H}$$

$$= \lim_{x \rightarrow \infty} \frac{e^x}{2e^{2x}} - \frac{e^{-x}}{2e^{2x}}$$

$$= \lim_{x \rightarrow \infty} \frac{1}{2e^x} - \frac{1}{2e^{3x}}$$

$$= 0$$

∴ Converges to 0.