Dawson College: **Calculus II**: 201-NYB-05-S3: Fall 2010

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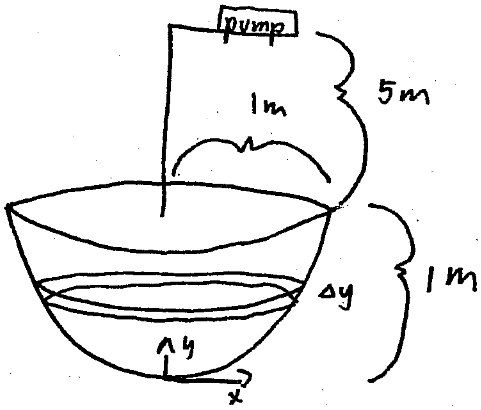
Quiz 11

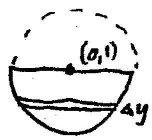
This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. *(5 marks)* A hemispherical tank is filled with liquid chocolate which has a density of $\rho = 1200\frac{kg}{m^3}$. If the tank is $2m$ across the top *(diameter)*, set up the integral that represent the work performed to empty half the tank of chocolate through a pipe that extends $5m$ above the top edge? $\left(g = 9.8\frac{m}{s^2}\right)$



Volume of slice:

$$\Delta V = \pi r^2 \Delta y$$
$$= \pi x^2 \Delta y$$
$$= \pi \left[1 - (y-1)^2\right] \Delta y$$



$$x^2 + (y-1)^2 = 1$$
$$x^2 = 1 - (y-1)^2$$

mass of slice:

$$\Delta m = \rho \Delta V$$
$$= 1200\pi \left[1 - (y-1)^2\right] \Delta y$$

force of slice:

$$\Delta F = \Delta m g$$
$$= 1200(9.8)\pi \left[1 - (y-1)^2\right] \Delta y$$
$$= 11760\pi \left[1 - (y-1)^2\right] \Delta y$$

distance of slice to pump: $d = 6 - y$

work to move slice: $\Delta W = \Delta F d$

$$= 11760\pi \left[1 - (y-1)^2\right](6-y)\Delta y$$

work:

$$W = \int_{1/2}^{1} 11760\pi \left[1 - (y-1)^2\right](6-y)\,dy$$

Question 2. §8.1 #5 *(2 marks)* Find a formula for the general term a_n of the sequence, assuming that the pattern of the first few terms continues.

$$\left\{1, -\frac{2}{3}, \frac{4}{9}, -\frac{8}{27}, \dots\right\}$$

$a_1 = 1$

$a_2 = -\frac{2}{3}$

$a_3 = \frac{4}{9}$

$a_4 = -\frac{8}{27}$

$\vdots$

$$a_n = (-1)^{n+1} \frac{2^{n-1}}{3^{n-1}}$$

Question 3. §8.1 #19 *(3 marks)* Determine whether the sequence converges or diverges. If it converges, find the limit.

$$\{n^2 e^{-n}\}$$

$a_n = n^2 e^{-n}$

Let $f(x) = x^2 e^{-x}$

then $\lim_{x \to \infty} x^2 e^{-x}$ I.F. $\infty \cdot 0$

$= \lim_{x \to \infty} \frac{x^2}{e^x}$ I.F. $\frac{\infty}{\infty}$

$= \lim_{x \to \infty} \frac{2x}{e^x}$ by $\hat{H}$ I.F. $\frac{\infty}{\infty}$

$= \lim_{x \to \infty} \frac{2}{e^x}$ by $\hat{H}$

$= 0$

$\therefore$ converges to 0.