Dawson College: **Calculus II**: 201-NYB-05-S3: Fall 2010

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Quiz 12

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. §8.2 #13 *(5 marks)* Determine whether the series is convergent or divergent. If it is convergent, find its sum.

$$\sum_{n=1}^{\infty} \frac{1+2^n}{3^n} = \sum_{n=1}^{\infty}\left[\frac{1}{3^n} + \frac{2^n}{3^n}\right]$$

$$= \sum_{n=1}^{\infty}\left(\frac{1}{3}\right)^n + \sum_{n=1}^{\infty}\left(\frac{2}{3}\right)^n \qquad \text{Let } a_n = \left(\frac{1}{3}\right)^n,\ b_n = \left(\frac{2}{3}\right)^n$$

$$= \sum_{n=1}^{\infty}\left(\frac{1}{3}\right)^n + a_0 - a_0 + \sum_{n=1}^{\infty}\left(\frac{2}{3}\right)^n + b_0 - b_0$$

$$= \sum_{n=0}^{\infty}\left(\frac{1}{3}\right)^n - \left(\frac{1}{3}\right)^0 + \sum_{n=0}^{\infty}\left(\frac{2}{3}\right)^n - \left(\frac{2}{3}\right)^0$$

$$= \frac{1}{1-1/3} - 1 + \frac{1}{1-2/3} - 1$$

$$= \frac{3}{2} + 3 - 2$$

$$= \frac{5}{2}$$

Question 2. §8.3 #13 *(5 marks)* Determine whether the series is convergent or divergent.

$$\sum_{n=1}^{\infty} ne^{-n}$$

Let $f(x) = xe^{-x}$

$f(x)$ is continuous on $[1,\infty)$
$f(x)$ is positive on $[1,\infty)$
$f(x)$ is decreasing on $[1,\infty)$ $\quad f'(x) = e^{-x} - xe^{-x}$

$$= \frac{1-x}{e^{x}} < 0 \quad \text{for } x>1$$

$$\int_1^{\infty} xe^{-x}\,dx = \lim_{b\to\infty} \int_1^{b} xe^{-x}\,dx$$

$u = x \quad du = dx$
$v = -e^{-x} \quad dv = e^{-x}\,dx$

$$= \lim_{b\to\infty} \left[[uv]_1^b - \int_1^b v\,du \right]$$

$$= \lim_{b\to\infty} \left[[-xe^{-x}]_1^b - \int_1^b -e^{-x}\,dx \right]$$

$$= \lim_{b\to\infty} \left[-be^{-b} + e^{-1} - [e^{-x}]_1^b \right]$$

$$= \lim_{b\to\infty} \left[\underbrace{\frac{-b}{e^{b}}}_{\text{I.F. } \frac{\infty}{\infty}} + \frac{1}{e} - \cancel{\frac{1}{e^{b}}}^{\,0} + \frac{1}{e} \right]$$

$$= \lim_{b\to\infty} \cancel{\frac{-1}{e^{b}}}^{\,0} + \frac{2}{e} \quad \text{by } \hat{H}$$

$$= \frac{2}{e}$$

$\therefore$ integral converges then by integral test the series converges.