Dawson College: **Calculus II**: 201-NYB-05-S3: Fall 2010

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Quiz 12

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. §8.2 #20 *(5 marks)* Determine whether the series is convergent or divergent. If it is convergent, find its sum.

$$\sum_{n=1}^{\infty} \frac{2}{n^2+4n+3} = \sum_{n=1}^{\infty} \frac{2}{(n+1)(n+3)}$$

$$= \sum_{n=1}^{\infty} \left[\frac{1}{n+1} - \frac{1}{n+3}\right]$$

$$\frac{2}{(n+1)(n+3)} = \frac{A}{n+1} + \frac{B}{n+3}$$

$$2 = A(n+3) + B(n+1)$$

Let $n=-1$

$$2 = A(-1+3) + B(-1+1)$$

$$1 = A$$

Let $n=-3$

$$2 = A(-3+3) + B(-3+1)$$

$$-1 = B$$

$$S_n = a_1 + a_2 + a_3 + a_4 + a_5 + \dots + a_{n-4} + a_{n-3} + a_{n-2} + a_{n-1} + a_n$$

$$= \left[\frac{1}{1+1} - \cancel{\frac{1}{1+3}}\right] + \left[\frac{1}{2+1} - \cancel{\frac{1}{2+3}}\right] + \left[\cancel{\frac{1}{3+1}} - \cancel{\frac{1}{3+3}}\right] + \left[\cancel{\frac{1}{4+1}} - \frac{1}{4+3}\right] + \left[\cancel{\frac{1}{5+1}} - \frac{1}{5+3}\right]$$

$$+ \dots + \left[\frac{1}{(n-4)+1} - \cancel{\frac{1}{(n-4)+3}}\right] + \left[\frac{1}{(n-3)+1} - \cancel{\frac{1}{(n-3)+3}}\right] + \left[\cancel{\frac{1}{(n-2)+1}} - \cancel{\frac{1}{(n-2)+3}}\right]$$

$$+ \left[\cancel{\frac{1}{(n-1)+1}} - \frac{1}{(n-1)+3}\right] + \left[\cancel{\frac{1}{n+1}} - \frac{1}{n+3}\right]$$

$$= \frac{1}{2} + \frac{1}{3} - \frac{1}{n+2} - \frac{1}{n+3}$$

$$S = \lim_{n\to\infty} S_n$$

$$= \lim_{n\to\infty} \left[\frac{1}{2} + \frac{1}{3} - \overset{0}{\cancel{\frac{1}{n+2}}} - \overset{0}{\cancel{\frac{1}{n+3}}}\right]$$

$$= \frac{5}{6}$$

Question 2. §8.3 #26 *(5 marks)* Determine whether the series is convergent or divergent.

$$\sum_{n=1}^{\infty} \frac{n+5}{\sqrt[3]{n^7+n^2}}$$

by inspection the series should behave like

$$\sum_{n=1}^{\infty} \frac{n}{\sqrt[3]{n^7}} = \sum_{n=1}^{\infty} \frac{n}{n^{7/3}} = \sum_{n=1}^{\infty} \frac{1}{n^{4/3}}$$ and should

converge since p-series where $p = 4/3 > 1$

Let $\sum b_n$ where $b_n = \frac{n}{\sqrt[3]{n^7}}$ be the test series which

converge. (see above)

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{\frac{n+5}{\sqrt[3]{n^7+n^2}}}{\frac{n}{\sqrt[3]{n^7}}}$$

$$= \lim_{n\to\infty} \frac{\sqrt[3]{n^7}\,(n+5)}{n\sqrt[3]{n^7+n^2}}$$

$$= \lim_{n\to\infty} \frac{(n+5)}{n} \cdot \frac{\sqrt[3]{n^7}}{\sqrt[3]{n^7+n^2}}$$

$$= \lim_{n\to\infty} \frac{n+5}{n} \cdot \lim_{n\to\infty} \sqrt[3]{\frac{n^7}{n^7+n^2}}$$

$$= 1 \cdot 1$$

$= 1 > 0$ and finite

$\therefore$ by limit comparison test the series converges since test series converges.