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Quiz 3

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. *(5 marks)* §5.1 #5a Estimate the area under the graph of $f(x) = 1 + x^2$ from $x = -1$ to $x = 2$ using three rectangles and right endpoints. Sketch the curve and approximating rectangles.
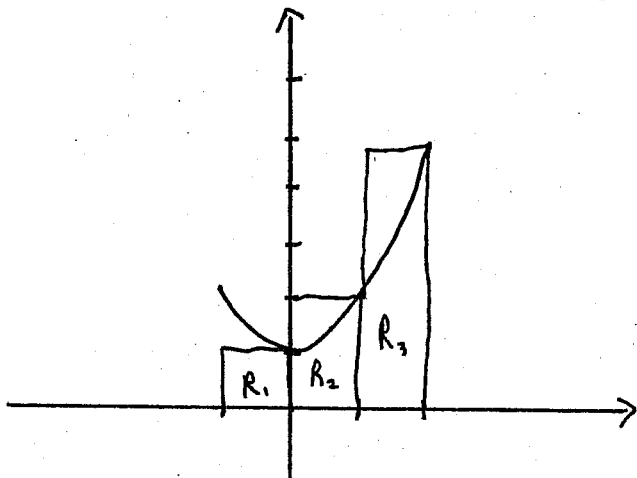
$$\text{Area} \approx R_1 + R_2 + R_3 \qquad \Delta x = 1$$
$$= f(x_1)\Delta x \qquad\qquad x_1 = 0$$
$$+ f(x_2)\Delta x \qquad\qquad x_2 = 1$$
$$+ f(x_3)\Delta x \qquad\qquad x_3 = 2$$
$$= f(0)\cdot 1 + f(1)\cdot 1 + f(2)\cdot 1$$
$$= 1\cdot 1 + 2\cdot 1 + 5\cdot 1$$
$$= 8$$

Question 2. *(5 marks)* §5.2 #21 Use only the definition of the definite integral to evaluate:

$$\int_0^2 (2 - x^2)\, dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x \quad \text{where} \quad f(x) = 2 - x^2, \quad \Delta x = \frac{b-a}{n} = \frac{2-0}{n} = \frac{2}{n}$$

$$x_i = a + i\Delta x = \frac{2i}{n}$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left(2 - \left(\frac{2i}{n}\right)^2\right)\frac{2}{n}$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{4}{n} - \frac{8i^2}{n^3}\right)$$

$$= \lim_{n\to\infty} \left[\frac{4}{n}\sum_{i=1}^{n} 1 - \frac{8}{n^3}\sum_{i=1}^{n} i^2\right]$$

$$= \lim_{n\to\infty} \left[\frac{4}{n}\, n - \frac{8}{n^3}\,\frac{n(n+1)(2n+1)}{6}\right]$$

$$= 4 - \frac{8}{6}\lim_{n\to\infty}\frac{(n+1)(2n+1)}{6n^2}$$

$$= 4 - \frac{4}{3}\lim_{n\to\infty}\frac{(n+1)}{n}\cdot\frac{(2n+1)}{n}$$

$$= 4 - \frac{4}{3}\cdot\frac{1}{1}\cdot 2$$

$$= 4 - \frac{8}{3}$$

$$= \frac{12 - 8}{3}$$

$$= \frac{4}{3}$$