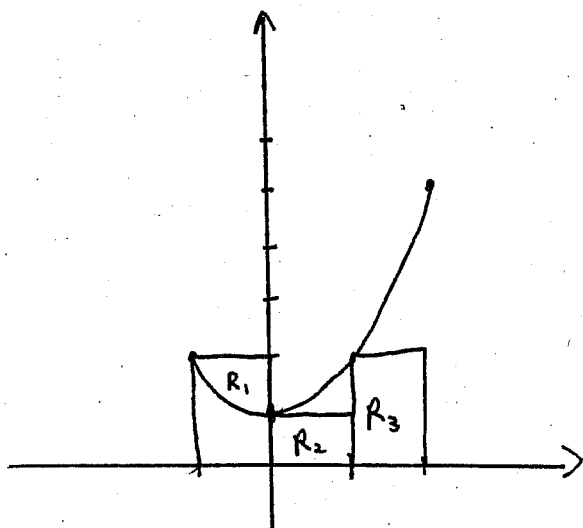


Quiz 3

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. (5 marks) §5.1 #5a Estimate the area under the graph of $f(x) = 1 + x^2$ from $x = -1$ to $x = 2$ using three rectangles and left endpoints. Sketch the curve and approximating rectangles.



$$\begin{aligned}
 \text{Area} &\approx R_1 + R_2 + R_3 & \Delta x &= 1 \\
 &= f(x_0)\Delta x + f(x_1)\Delta x & x_0 &= -1 \\
 &+ f(x_2)\Delta x & x_1 &= 0 \\
 &= f(-1) \cdot 1 + f(0) \cdot 1 & x_2 &= 1 \\
 &+ f(1) \cdot 1 \\
 &= 2 \cdot 1 + 1 \cdot 1 + 2 \cdot 1 \\
 &= 5
 \end{aligned}$$

Question 2. (5 marks) §5.2 #22 Use only the definition of the definite integral to evaluate:

$$\begin{aligned}
 \int_0^5 (1+2x^3) dx &= \lim_{n \rightarrow \infty} \sum_{i=1}^n f(x_i) \Delta x \quad \text{where } f(x) = 1+2x^3, \Delta x = \frac{b-a}{n} = \frac{5-0}{n} = \frac{5}{n} \\
 &= \lim_{n \rightarrow \infty} \sum_{i=1}^n \left(1 + 2 \left(\frac{5i}{n} \right)^3 \right) \frac{5}{n} & x_i &= a + i\Delta x = \frac{5i}{n} \\
 &= \lim_{n \rightarrow \infty} \frac{5}{n} \sum_{i=1}^n \left(1 + \frac{250i^3}{n^3} \right) \\
 &= \lim_{n \rightarrow \infty} \frac{5}{n} \left[\sum_{i=1}^n 1 + \frac{250}{n^3} \sum_{i=1}^n i^3 \right] \\
 &= \lim_{n \rightarrow \infty} \frac{5}{n} \left[n + \frac{125}{n^3} \frac{n^2(n+1)^2}{4} \right] \\
 &= \lim_{n \rightarrow \infty} \left[5 + \frac{625}{2} \frac{n^2}{n^3} \frac{(n+1)^2}{n^2} \right] \\
 &= \frac{635}{2}
 \end{aligned}$$