Dawson College: **Calculus II**: 201-NYB-05-S3: Fall 2010

Name: Y. Lamontagne
Student ID: _____

Quiz 6

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. §6.2 #18 *(5 marks)* Evaluate the definite integral:

$$\int_0^{\pi/2} \sec^4\left(\frac{t}{2}\right) dt = \int_0^{\pi/2} \sec^2\left(\frac{t}{2}\right)\sec^2\left(\frac{t}{2}\right) dt$$

$$= \int_0^{\pi/2} \sec^2\left(\frac{t}{2}\right)\left(1+\tan^2\left(\frac{t}{2}\right)\right) dt$$

$u = \tan\left(\frac{t}{2}\right)$

$du = \sec^2\left(\frac{t}{2}\right)\frac{1}{2}\,dt$

$2du = \sec^2\left(\frac{t}{2}\right)dt$

$u(0) = \tan 0 = 0$

$u\left(\frac{\pi}{2}\right) = \tan\left(\frac{\frac{\pi}{2}}{2}\right) = 1$

$$= \int_0^1 (1+u^2)(2)\,du$$

$$= 2\int_0^1 (1+u^2)\,du$$

$$= 2\left[u + \frac{u^3}{3}\right]_0^1$$

$$= 2\left[1 + \frac{1}{3}\right]$$

$$= 2\left(\frac{4}{3}\right)$$

$$= \frac{8}{3}$$

Question 2. §6.2 #45 *(5 marks)* Evaluate the indefinite integral:

$$\int \frac{1}{\sqrt{x^2+16}}\,dx = \int \frac{1}{\sqrt{(4\tan\theta)^2+16}}\,4\sec^2\theta\,d\theta$$

$x = 4\tan\theta$

$dx = 4\sec^2\theta\,d\theta$

$$= \int \frac{1}{\sqrt{16\tan^2\theta+16}}\,4\sec^2\theta\,d\theta$$

$$= \int \frac{1}{\sqrt{16(\tan^2\theta+1)}}\,4\sec^2\theta\,d\theta$$

$$= \int \frac{1}{\sqrt{16\sec^2\theta}}\,4\sec^2\theta\,d\theta$$

$$= \int \frac{1}{4\sec\theta}\,4\sec^2\theta\,d\theta$$

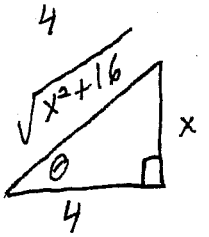
$$= \int \sec\theta\,d\theta$$

$$= \ln|\sec\theta + \tan\theta| + C$$

$$= \ln\left|\frac{\sqrt{x^2+16}}{4} + \frac{x}{4}\right| + C$$

note: $x = 4\tan\theta$

$\frac{x}{4} = \tan\theta$



$\sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{\sqrt{x^2+16}}{4}$