

## Quiz 6

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

**Question 1.** §6.2 #23 (5 marks) Evaluate the definite integral:

$$\int_0^{\pi/3} \tan^5 x \sec^4 x dx = \int_0^{\pi/3} \tan^5 x \sec^2 x \sec^2 x dx$$

$$= \int_0^{\pi/3} \tan^5 x (1 + \tan^2 x) \sec^2 x dx$$

$$= \int_0^{\sqrt{3}} u^5 (1 + u^2) du$$

$$= \int_0^{\sqrt{3}} u^5 + u^7 du$$

$$= \left[ \frac{u^6}{6} + \frac{u^8}{8} \right]_0^{\sqrt{3}}$$

$$= \left[ \frac{(\sqrt{3})^6}{6} + \frac{(\sqrt{3})^8}{8} \right]$$

$$= \left[ \frac{3^3}{6} + \frac{3^4}{8} \right]$$

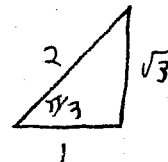
$$= \left[ \frac{27}{6} + \frac{81}{8} \right]$$

$$= \left[ \frac{9}{2} + \frac{81}{8} \right]$$

$$= \left[ \frac{36}{8} + \frac{81}{8} \right]$$

$$= \frac{117}{8}$$

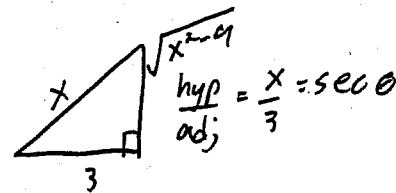
$$\begin{aligned} u &= \tan x \\ du &= \sec^2 x dx \\ u(0) &= \tan 0 = 0 \\ u\left(\frac{\pi}{3}\right) &= \tan \frac{\pi}{3} = \sqrt{3} \end{aligned}$$



Question 2. §6.2 #44 (5 marks) Evaluate the indefinite integral:

$$\int \frac{1}{x^2 \sqrt{x^2 - 9}} dx \quad x = 3 \sec \theta \quad \Rightarrow$$

$$dx = 3 \sec \theta \tan \theta d\theta$$



$$= \int \frac{1}{(3 \sec \theta)^2 \sqrt{(3 \sec \theta)^2 - 9}} 3 \sec \theta \tan \theta d\theta$$

$$= \frac{1}{3} \int \frac{1}{\sec \theta \sqrt{9 \sec^2 \theta - 9}} \tan \theta d\theta$$

$$= \frac{1}{3} \int \frac{1}{\sec \theta \sqrt{9(\sec^2 \theta - 1)}} \tan \theta d\theta$$

$$= \frac{1}{3} \int \frac{1}{\sec \theta \sqrt{9 \tan^2 \theta}} \tan \theta d\theta$$

$$= \frac{1}{3} \int \frac{1}{\sec \theta 3 \tan \theta} \tan \theta d\theta$$

$$\frac{1}{9} \int \cos \theta d\theta$$

$$\frac{1}{9} \sin \theta + C$$

$$\frac{1}{9} \cdot \frac{\sqrt{x^2 - 9}}{x} + C$$