

Quiz 7

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. §6.3 #18 (5 marks) Evaluate the indefinite integral:

$$\int \frac{x^2 + 2x - 1}{x^3 - x} dx = \int \frac{1}{x} + \frac{1}{x-1} - \frac{1}{x+1} dx = \ln|x| + \ln|x-1| - \ln|x+1| + C$$

$$\frac{x^2 + 2x - 1}{x(x^2 - 1)} = \frac{x^2 + 2x - 1}{x(x-1)(x+1)} = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{x+1}$$

$$\frac{(x^2 + 2x - 1)\cancel{x}\cancel{(x-1)}\cancel{(x+1)}}{x\cancel{(x-1)}\cancel{(x+1)}} = \frac{A\cancel{x}\cancel{(x-1)}\cancel{(x+1)}}{x} + \frac{B(x)\cancel{(x-1)}\cancel{(x+1)}}{x-1} + \frac{C(x)\cancel{(x-1)}\cancel{(x+1)}}{x+1}$$

$$x^2 + 2x - 1 = A(x-1)(x+1) + Bx(x+1) + Cx(x-1)$$

$$\text{Let } x=0: (0)^2 + 2(0) - 1 = A(-1)(1) + B(0)(1) + C(0)(-1)$$

$$-1 = -A$$

$$A = 1$$

$$\text{Let } x=1: (1)^2 + 2(1) - 1 = A(1-1)(1+1) + B(1)(1+1) + C(1)(1-1)$$

$$2 = 2B$$

$$1 = B$$

$$\text{Let } x=-1: (-1)^2 + 2(-1) - 1 = A(-1-1)(-1+1) + B(-1)(-1+1) + C(-1)(-1-1)$$

$$-2 = 2C$$

$$-1 = C$$

Question 2. (5 marks) Use the Trapezoidal Rule to approximate the given integral with $n = 4$

$$\int_0^2 x \sin x dx \approx \frac{\Delta x}{2} \left[f(x_0) + 2f(x_1) + 2f(x_2) + 2f(x_3) + f(x_4) \right]$$

$$\Delta x = \frac{b-a}{n} = \frac{2-0}{4} = \frac{1}{2} = \frac{1}{4} \left[f(0) + 2f\left(\frac{1}{2}\right) + 2f(1) + 2f\left(\frac{3}{2}\right) + f(2) \right]$$

$$x_i = a + i\Delta x$$

$$x_i = \frac{i}{2}$$

$$x_0 = 0$$

$$x_1 = \frac{1}{2}$$

$$x_2 = 1$$

$$x_3 = \frac{3}{2}$$

$$x_4 = 2$$

$$= \frac{1}{4} \left[0 \sin 0 + 2\left(\frac{1}{2}\right) \sin\left(\frac{1}{2}\right) + 2(1) \sin 1 + 2\left(\frac{3}{2}\right) \sin\left(\frac{3}{2}\right) + 2 \sin 2 \right]$$

$$= \frac{1}{4} \left[\sin\left(\frac{1}{2}\right) + 2 \sin(1) + 3 \sin\left(\frac{3}{2}\right) + 2 \sin 2 \right]$$

$$\approx 1.743361830$$