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Quiz 7

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. §6.3 #31 *(5 marks)* Evaluate the indefinite integral:

$$\int \frac{1}{x^4 - x^2}\,dx = \int \frac{-1}{x^2} + \frac{1/2}{x-1} - \frac{1/2}{x+1}\,dx = \frac{1}{x} + \frac{1}{2}\ln|x-1| - \frac{1}{2}\ln|x+1| + C$$

$$\frac{1}{x^4 - x^2} = \frac{1}{x^2(x^2-1)} = \frac{1}{x^2(x-1)(x+1)} = \frac{Ax+B}{x^2} + \frac{C}{x-1} + \frac{D}{x+1}$$

$$\frac{1\,x^2(x-1)(x+1)}{x^2(x-1)(x+1)} = \frac{(Ax+B)x^2(x-1)(x+1)}{x^2} + \frac{C\,x^2(x-1)(x+1)}{x-1} + \frac{D\,x^2(x-1)(x+1)}{x+1}$$

$$1 = (Ax+B)(x-1)(x+1) + C\,x^2(x+1) + D\,x^2(x-1)$$

Let $x=0$: $1 = (A(0)+B)(0-1)(0+1) + C(0)^2(0+1) + D(0)^2(0-1)$

$1 = -B$

$-1 = B$

Let $x=1$: $1 = (A(1)+B)(1-1)(1+1) + C(1)^2(1+1) + D(1)^2(1-1)$

$C = 1/2$

Let $x=-1$: $1 = (A(-1)+B)(-1-1)(-1+1) + C(-1)^2(-1+1) + D(-1)^2(-1-1)$

$D = -1/2$

Let $x=2$: $1 = (A(2)-1)(2-1)(2+1) + \frac{1}{2}(2)^2(2+1) + \left(\frac{-1}{2}\right)(2)^2(2-1)$

$1 = 6A - 3 + 6 - 2$

$0 = A$

Question 2. *(5 marks)* Use Simpson's Rule to approximate the given integral with $n = 4$

$$\int_0^1 x\cos x\,dx \approx \frac{\Delta x}{3}\left[f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + f(x_4)\right]$$

$\Delta x = \frac{b-a}{n} = \frac{1-0}{4} = \frac{1}{4}$

$x_i = a + i\Delta x$

$x_i = \frac{i}{4}$

$x_0 = 0$

$x_1 = 1/4$

$x_2 = 2/4$

$x_3 = 3/4$

$x_4 = 1$

$$= \frac{1}{12}\left[f(0) + 4f(1/4) + 2f(1/2) + 4f(3/4) + f(1)\right]$$

$$= \frac{1}{12}\left[0\cos 0 + 4\left(\frac{1}{4}\right)\cos\left(\frac{1}{4}\right) + 2\left(\frac{1}{2}\right)\cos\left(\frac{1}{2}\right) + 4\left(\frac{3}{4}\right)\cos\left(\frac{3}{4}\right) + 1\cos(1)\right]$$

$$= \frac{1}{12}\left[\cos\left(\frac{1}{4}\right) + \cos\left(\frac{1}{2}\right) + 3\cos\left(\frac{3}{4}\right) + \cos(1)\right]$$

$$= 0.3818\ 219914$$