Name: Y LAMONTAGNE
Student ID:

Quiz 8

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. §3.7 #32 *(5 marks)* Find the limit:

$y = \lim_{x\to 0^+} (\tan 2x)^x$ I.F. 0^0

$\ln y = \ln \lim_{x\to 0^+} (\tan 2x)^x$

$\ln y = \lim_{x\to 0^+} \ln (\tan 2x)^x$

$\ln y = \lim_{x\to 0^+} x \ln(\tan 2x)$ I.F. $0\cdot(-\infty)$

$\ln y = \lim_{x\to 0^+} \frac{\ln(\tan 2x)}{1/x}$ I.F. $\frac{-\infty}{\infty}$

$\ln y = \lim_{x\to 0^+} \frac{\frac{1}{\tan 2x}(\sec^2 2x)(2)}{-1/x^2}$ by $\hat{H}$

$\ln y = \lim_{x\to 0^+} -\frac{\cancel{\cos 2x}}{\sin 2x}\cdot\frac{1}{\cos^{\cancel{2}} 2x}\cdot 2\cdot x^2$

$\ln y = \lim_{x\to 0^+} \frac{-2x^2}{\sin 2x \cos 2x}$ I.F. $\frac{0}{0}$

$\ln y = \lim_{x\to 0^+} \frac{-4x}{2\cos 2x\cos 2x - 2\sin 2x \sin 2x}$ by $\hat{H}$

$\ln y = 0$

$e^{\ln y} = e^0$

$y = 1$

$\therefore \lim_{x\to 0^+} (\tan 2x)^x = 1$

Question 2. §6.6 #20 *(5 marks)* Determine whether the integral is convergent or divergent. Evaluate if convergent.

$\int_1^\infty \frac{\ln x}{x^3}dx = \lim_{b\to\infty} \int_1^b \frac{\ln x}{x^3}dx$

$= \lim_{b\to\infty} \left[[uv]_1^b - \int_1^b v\,du \right]$

$u = \ln x$ $\quad du = \frac{1}{x}dx$

$v = \frac{-1}{2x^2}$ $\quad dv = \frac{1}{x^3}dx$

$= \lim_{b\to\infty} \left[\left[\frac{-\ln x}{2x^2}\right]_1^b - \int_1^b \frac{-1}{2x^3}dx \right]$

$= \lim_{b\to\infty} \left[\left[\frac{-\ln b}{2b^2} - \underbrace{\frac{-\ln 1}{2(1)^2}}_{0}\right] - \left[\frac{1}{4x^2}\right]_1^b \right]$

I.F. $\frac{\infty}{\infty}$

$= \lim_{b\to\infty} \frac{-\cancel{\frac{1}{b}}}{\cancel{4b}} \to 0 - \frac{1}{\cancel{4b^2}} \to 0 + \frac{1}{4}$

$= \frac{1}{4}$