Dawson College: Calculus II: 201-NYB-05-S3: Fall 2010

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Quiz 8

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your ɔrk, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. §3.7 #35 *(5 marks)* Evaluate the indefinite integral:

$y = \lim_{x\to0^+} (\cos x)^{1/x^2}$ I.F 1^∞

$\ln y = \ln \lim_{x\to0^+} (\cos x)^{1/x^2}$

$\ln y = \lim_{x\to0^+} \ln (\cos x)^{1/x^2}$

$\ln y = \lim_{x\to0^+} \frac{1}{x^2} \ln \cos x$ I.F $\frac{0}{0}$

$\ln y = \lim_{x\to0^+} \frac{\frac{-\sin x}{\cos x}}{2x}$ by $\hat{H}$

$\ln y = \lim_{x\to0^+} \frac{-\sin x}{2x\cos x}$ I.F. $\frac{0}{0}$

$\ln y = \lim_{x\to0^+} \frac{-\cos x}{2\cos x + 2x(-\sin x)}$ by $\hat{H}$

$\ln y = \frac{-1}{2}$

$e^{\ln y} = e^{-1/2}$

$y = e^{-1/2}$

$\therefore \lim_{x\to0^+} (\cos x)^{1/x^2} = e^{-1/2}$

Question 2. §6.6 #32 *(5 marks)* Determine whether the integral is convergent or divergent. Evaluate if convergent.

$\int_0^1 \frac{\ln x}{\sqrt{x}}dx = \lim_{a\to0^+} \int_a^1 \frac{\ln x}{\sqrt{x}}$

$u = \ln x \qquad du = \frac{1}{x}dx$

$v = 2\sqrt{x} \qquad dv = \frac{1}{\sqrt{x}}dx$

$= \lim_{a\to0^+} \left[[uv]_a^1 - \int_a^1 v\,du \right]$

$= \lim_{a\to0^+} \left[[2\sqrt{x}\ln x]_a^1 - \int_a^1 2\sqrt{x}\left(\frac{1}{x}\right)dx \right]$

$= \lim_{a\to0^+} \left[[2\sqrt{1}\ln 1] - [2\sqrt{a}\ln a] - [4\sqrt{x}]_a^1 \right]$

$= \lim_{a\to0^+} \left[-2\sqrt{a}\ln a - 4 + 4\sqrt{a}^{\to 0} \right]$ I.F. $0\cdot(-\infty)$

$= \lim_{a\to0^+} \frac{-2\ln a}{1/\sqrt{a}} - 4$ I.F. $\frac{-\infty}{\infty}$

$= \lim_{a\to0^+} \frac{-2/a}{\frac{1}{2}a^{-3/2}} - 4$ by $\hat{H}$

$= \lim_{a\to0^+} -4a^{1/2} - 4$

$= -4$