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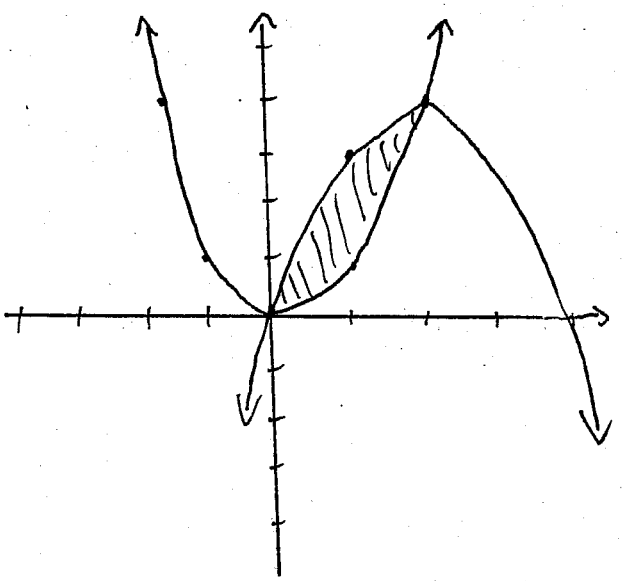
Quiz 9

This quiz is graded out of 15 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. §7.1 #10 *(5 marks)* Sketch the region enclosed by the given curves. Then find the area of the region.

$y = x^2,\ y = 4x - x^2$



intersection: $x^2 = 4x - x^2$

$0 = 4x - 2x^2$

$0 = 2x(2-x)$

$x = 0$ $\quad$ $x = 2$

x-int: $0 = 4x - x^2$

$0 = x(4-x)$

$$A = \int_0^2 4x - x^2 - x^2\,dx$$

$$= \int_0^2 4x - 2x^2\,dx$$

$$= \left[2x^2 - \frac{2x^3}{3}\right]_0^2$$

$$= 2\cdot 2^2 - 2\cdot\frac{2^3}{3} = 8 - \frac{16}{3} = \frac{8}{3}$$

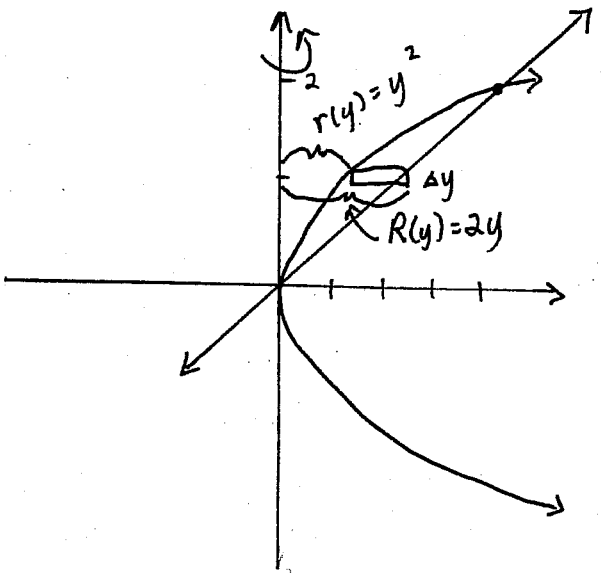
Question 2. §6.6 #20 *(5 marks)* Determine whether the integral is convergent or divergent. Evaluate if convergent.

$$\int_1^\infty \frac{\ln x}{x^3}\,dx$$

see quiz 8

Question 3. §7.2 #7 *(5 marks)* Find the volume of the solid obtained by rotating the region bounded by the given curves about the specified line.

$y^2 = x,\ x = 2y;$ about the y-axis



Intersection:

$y^2 = 2y$

$0 = y^2 - 2y$

$0 = y(y-2)$

$y = 0 \qquad y = 2$

rep. element:

$\Delta V = \pi\left[(R(y))^2 - (r(y))^2\right]\Delta y$

$= \pi\left[(2y)^2 - (y^2)^2\right]\Delta y$

$= \pi\left[4y^2 - y^4\right]\Delta y$

$\therefore\ V = \int_0^2 \pi\left[4y^2 - y^4\right]dy$

$= \pi\int_0^2 4y^2 - y^4\,dy$

$= \pi\left[\frac{4y^3}{3} - \frac{y^5}{5}\right]_0^2$

$= \pi\left[\frac{4\cdot 2^3}{3} - \frac{2^5}{5}\right]$

$= \pi\left[\frac{32}{3} - \frac{32}{5}\right]$

$= \frac{64}{15}\pi$