

Name:

Student ID:

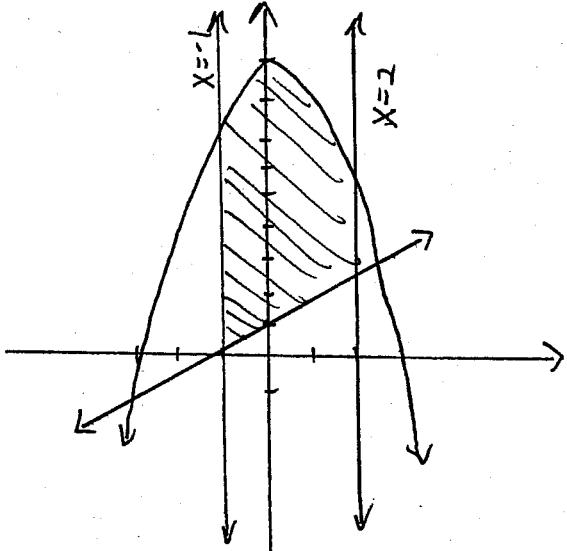
Y. Lamontagne

Quiz 9

This quiz is graded out of 15 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. §7.1 #5 (5 marks) Sketch the region enclosed by the given curves. Then find the area of the region.

$$y = x + 1, \quad y = 9 - x^2, \quad x = -1, \quad x = 2$$



$$\begin{aligned}
 \text{Area} &= \int_{-1}^2 (9 - x^2) - (x + 1) \, dx \\
 &= \int_{-1}^2 -x^2 - x + 8 \, dx \\
 &= \left[-\frac{x^3}{3} - \frac{x^2}{2} + 8x \right]_{-1}^2 \\
 &= -\frac{(2)^3}{3} - \frac{2^2}{2} + 8(2) - \left[-\frac{(-1)^3}{3} - \frac{(-1)^2}{2} + 8(-1) \right] \\
 &= -\frac{8}{3} - \frac{4}{2} + 16 - \left[\frac{1}{3} - \frac{1}{2} - 8 \right] \\
 &= \frac{39}{2}
 \end{aligned}$$

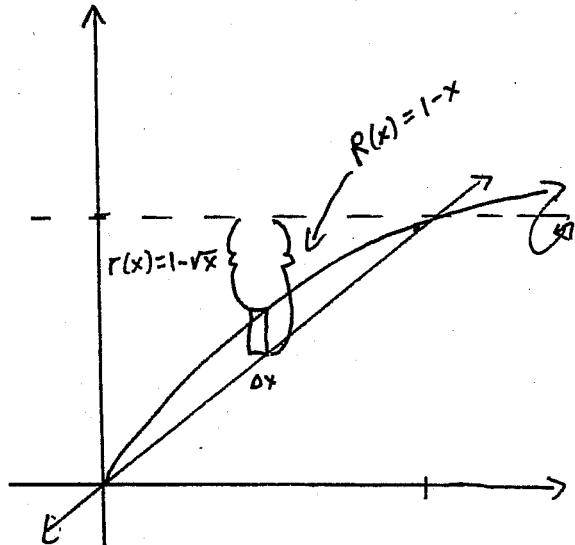
Question 2. §6.6 #32 (5 marks) Determine whether the integral is convergent or divergent. Evaluate if convergent.

$$\int_0^1 \frac{\ln x}{\sqrt{x}} \, dx$$

See Quiz 8

Question 3. §7.2 #9 (5 marks) Find the volume of the solid obtained by rotating the region bounded by the given curves about the specified line.

$$y = x, \quad y = \sqrt{x}; \quad \text{about } y = 1$$



Intersection:

$$(x) = (\sqrt{x})^2$$

$$x^2 = x$$

$$x^2 - x = 0$$

$$x(x-1) = 0$$

$$\begin{matrix} / \\ x=0 \end{matrix} \quad \begin{matrix} \backslash \\ x=1 \end{matrix}$$

rep element:

$$\Delta V = \pi [(R(x))^2 - (r(x))^2] \Delta x$$

$$= \pi [(1-x)^2 - (1-\sqrt{x})^2] \Delta x$$

$$= \pi [1 - 2x + x^2 - 1 + 2\sqrt{x} - x] \Delta x$$

$$= \pi [2\sqrt{x} - 3x + x^2] \Delta x$$

$$V = \int_0^1 \pi [2\sqrt{x} - 3x + x^2] dx$$

$$= \pi \left[\frac{2x^{3/2}}{\frac{3}{2}} - \frac{3x^2}{2} + \frac{x^3}{3} \right]_0^1$$

$$= \pi \left[\frac{4}{3} - \frac{3}{2} + \frac{1}{3} \right]$$

$$= \frac{\pi}{6}$$