

Name: _____
Student ID: _____

Test 1

This test is graded out of 40 marks. No books, notes, graphing calculators or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Formulae:

$$\sum_{i=1}^n c = cn \text{ where } c \text{ is a constant}$$
$$\sum_{i=1}^n i = \frac{n(n+1)}{2}$$
$$\sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6}$$
$$\sum_{i=1}^n i^3 = \frac{n^2(n+1)^2}{4}$$

Question 1. (5 marks) Evaluate using the definition of the definite integral

$$\int_1^3 -x^2 - x + 1 \, dx.$$

Question 2. (2 marks) Evaluate the indefinite integral:

$$\int \tan x - \csc x + \frac{1}{\sqrt{11-x^2}} dx$$

Question 3. (4 marks) Evaluate the definite integral:

$$\int_1^e \frac{(x-1)(x+1)^2}{x} + \frac{2}{x} dx$$

Question 4. (4 marks) Evaluate the definite integral:

$$\int_{\pi/4}^{\pi/3} \frac{\sin \theta + \sin \theta \tan^2 \theta}{\sec^2 \theta} d\theta$$

Question 5. (5 marks) A super galactic space ship is traveling towards Pluto at a speed of

$$v(t) = \frac{1}{\pi t^2 + 16}$$

Terameters per hour. What distance was traveled after 4 hours? What is the average speed of the ship after 4 hours of travel?

Question 6. (5 marks + 1 bonus mark to simplify completely) Compute the derivative:

$$\frac{d}{dx} \left[\int_{\arcsin 3x}^{\arctan 2x} \sin^{100} t \tan^{1001} t \, dt \right]$$

Question 7. (4 marks) Evaluate the indefinite integral:

$$\int x\sqrt[3]{x-1} dx$$

Question 8. (4 marks) Estimate the area under the graph of $f(x) = \arctan x$ from $x = \frac{1}{4}$ to $x = \frac{5}{4}$ using two rectangles and the Midpoint Rule. Sketch the curve and approximating rectangles.

Question 9. (4 marks) Evaluate the indefinite integral:

$$\int \frac{1}{e^{\tan z} \cos^2 z} dz$$

Question 10. (3 marks) Prove (without using the Fundamental Theorem of Calculus):

$$\int_a^b c dx = c(b-a)$$

where c is a constant.

Bonus Question. (3 marks)

Evaluate the following limit:

$$\lim_{\zeta \rightarrow 0} \frac{\int_{\pi}^{\arctan(x+\zeta)} \tan z \sin^{101} z \, dz - \int_{\pi}^{\arctan x} \tan z \sin^{101} z \, dz}{\zeta}$$