

Name: _____
Student ID: _____

Test 3

This test is graded out of 40 marks. No books, notes, graphing calculators or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. (5 marks) Set up, but do not evaluate, an integral for the volume of the solid obtained by rotating the region bounded by the given curves about the specified line.

$$y = x + 3, y = \sqrt{x + 1} + 3, y = 3 \quad \text{about } y = -2$$

Question 2. (5 marks) Find the length of the curve.

$$y = \ln(\sec x), \quad 0 \leq x \leq \frac{\pi}{4}$$

Question 3. (5 marks) A hemispherical tank is filled with a liquid which has a density of $\rho = 1000 \frac{\text{kg}}{\text{m}^3}$. If the tank is 4m across the top (*diameter*), set up the integral that represent the work performed to empty the tank through a pipe that extends 9m above the top edge?
($g = 9.8 \frac{\text{m}}{\text{s}^2}$)

Question 4. (2 marks) Find a formula for the general term a_n of the sequence, assuming that the pattern of the first few terms continues.

$$\left\{ 1, -\frac{1}{2}, \frac{2}{3}, -\frac{6}{4}, \frac{24}{5}, \dots \right\}$$

Question 5. Suppose that you and Yann have infinite life. If Yann gives you 20 grams of Krypton (not to be confused with Kryptonite) on the first day, then $\frac{3}{5}$ of that amount on the second day, then on the third day, you are given $\frac{3}{5}$ of the amount of the second day, and this process continues forever.

- a. (1 mark) Will Yann give you an infinite amount of Krypton?
- b. (2 marks) What is the amount of Krypton given to you by the process described above?
- c. (bonus 1 mark) Does Yann have an infinite amount of Krypton, Justify.

Question 6. (5 marks) Determine whether each of the following series converges or diverges. Justify your answer.

a. (5 marks)

$$\sum_{n=10}^{\infty} \frac{n\sqrt[3]{n^9 + 3n^4 + 4}}{\sqrt{n^9 + n^4}}$$

b. (5 marks)

$$\sum_{n=1}^{\infty} \frac{e^{-n^{1/5}}}{n^{4/5}}$$

Question 7. (5 marks) Find the sum of the following series if it converges or show it diverges.

$$\sum_{n=1}^{\infty} \cos\left(2\pi - \frac{1}{n}\right)$$

Question 8. (5 marks) Find the Taylor Polynomial of order 2 of $f(x) = \arctan(5x)$ at $x = \frac{1}{5}$.

Bonus Question. (3 marks)

Prove: If

$$\sum_{n=k}^{\infty} a_n$$

converges then

$$\lim_{n \rightarrow \infty} a_n = 0.$$