Dawson College: Introduction to Applied Mathematics: 201-912-DW-S1: Fall 2011

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Test 2

This test is graded out of 48 marks. No books, notes, graphing calculators or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. *(3 marks) (use the correct number of significant figures)* This problem outlines how Yann of Montreal approximated the diameter of the earth in 2011. By observing the shadow of the sun at noon he recognized that the town of Leh was approximately 3.6° north of the town of Manali. He cycled and measured a distance of 405 *km* between Manali and Leh. Based on these figures, what is the circumference of the Earth in Kilometers.

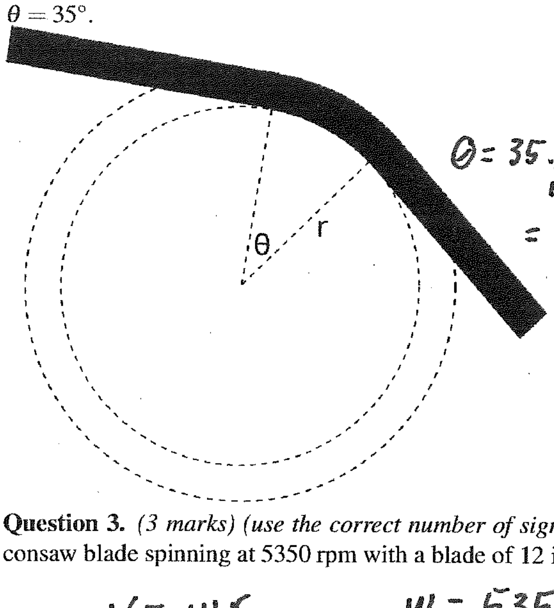
3.6° is to 360° as cycling 405 Km is to cycling the circumference of the Earth

$$\frac{3.6^\circ}{360^\circ} = \frac{405}{C}$$

$$C = \frac{360^\circ(405)}{3.6^\circ}$$

$$C = 41\,000 \text{ Km}$$

Question 2. *(3 marks)* Compute the area of the curved portion of the sidewalk given that the sidewalk*(shaded below)* is 2*m* wide, $r = 10$, $\theta = 35°$.



$$\theta = 35 \cdot \frac{\pi}{180} \text{ rad} = \frac{7\pi}{36} \text{ rad}$$

Area = Area of larger sector − Area of smaller sector

$$= \frac{1}{2}\theta r_1^2 - \frac{1}{2}\theta r_2^2$$

$$= \frac{1}{2}\frac{7\pi}{36}12^2 - \frac{1}{2}\frac{7\pi}{36}10^2$$

$$= \frac{77\pi}{18} \doteq 13 \text{ m}^2$$

Question 3. *(3 marks) (use the correct number of significant figures)* What is the linear velocity in meters per seconds of the tip of a consaw blade spinning at 5350 rpm with a blade of 12 inches in diameter?

$$w = 5350\frac{\text{rev}}{\text{min}} \cdot \frac{2\pi \text{ rad}}{1 \text{ rev}} \cdot \frac{1 \text{ min}}{60 \text{ s}} = 560.\frac{\text{rad}}{\text{s}}$$

$$r = \frac{12 \text{ in}}{2} \cdot \frac{2.54 \text{ cm}}{1 \text{ in}} = \frac{30. \text{cm}}{2} = \frac{0.30 \text{ m}}{2}$$

$$V = wr$$

$$= 560\left(\frac{0.30}{2}\right)$$

$$= \frac{170}{2}\frac{\text{m}}{\text{s}}$$

$$= 85 \text{ m/s}$$

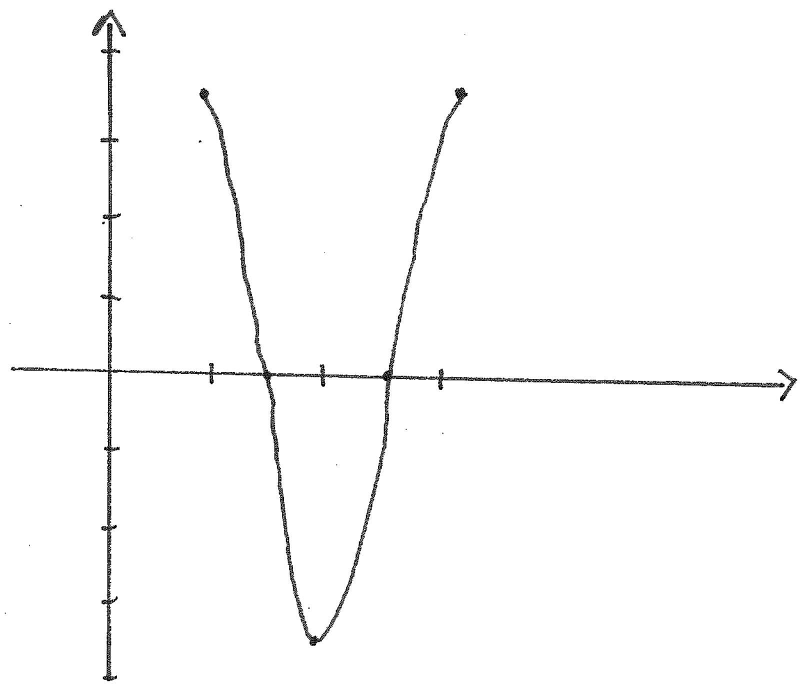
Question 4. *(5 marks)* Sketch one period of the graph of $f(x) = 3.5\cos(\pi x - \pi)$ by finding and using its key points (i.e. multiple of period/4 + displacement).

amplitude $= |a| = 3.5$

period $= \frac{2\pi}{b} = \frac{2\pi}{\pi} = 2$

displacement $= \frac{-c}{b} = \frac{-(-\pi)}{\pi} = 1$

x	y
$\frac{2}{4}(0) + 1 = 1$	$3.5\cos(\pi(1) - \pi) = 3.5$
$\frac{2}{4}(1) + 1 = \frac{3}{2}$	$3.5\cos(\pi \frac{3}{2} - \pi) = 0$
$\frac{2}{4}(2) + 1 = 2$	$3.5\cos(\pi 2 - \pi) = -3.5$
$\frac{2}{4}(3) + 1 = \frac{5}{2}$	$3.5\cos(\pi \frac{5}{2} - \pi) = 0$
$\frac{2}{4}(4) + 1 = 3$	$3.5\cos(3\pi - \pi) = 3.5$



Question 5. *(5 marks)* Use Cramer's rule to solve for x, y.

$$\begin{aligned} 3x + 2y &= 1 \\ 4x + y &= 0 \end{aligned}$$

$|A| = \begin{vmatrix} 3 & 2 \\ 4 & 1 \end{vmatrix} = 3 - 8 = -5$

$|A_1| = \begin{vmatrix} 1 & 2 \\ 0 & 1 \end{vmatrix} = 1$

$|A_2| = \begin{vmatrix} 3 & 1 \\ 4 & 0 \end{vmatrix} = -4$

$x = \frac{|A_1|}{|A|} = \frac{1}{-5}$

$y = \frac{|A_2|}{|A|} = \frac{-4}{-5} = \frac{4}{5}$

Question 6. *(5 marks)* Use Cramer's rule to solve for y without solving for x, z.

$$\begin{aligned} 3x + 2y + 2z &= 1 \\ 4x + y &= 0 \\ 7x + 3y - z &= 3 \end{aligned}$$

$$|A| = \begin{vmatrix} 3 & 2 & 2 \\ 4 & 1 & 0 \\ 7 & 3 & -1 \end{vmatrix} = 3\begin{vmatrix} 1 & 0 \\ 3 & -1 \end{vmatrix} - 2\begin{vmatrix} 4 & 0 \\ 7 & -1 \end{vmatrix} + 2\begin{vmatrix} 4 & 1 \\ 7 & 3 \end{vmatrix}$$

$$= 3\left[(1)(-1)-3(0)\right] - 2\left[4(-1)-7(0)\right] + 2\left[4(3)-1(7)\right]$$

$$= 3[-1] - 2[-4] + 2[5] = 15$$

$$|A_2| = \begin{vmatrix} 3 & 1 & 2 \\ 4 & 0 & 0 \\ 7 & 3 & -1 \end{vmatrix} = 3\begin{vmatrix} 0 & 0 \\ 3 & -1 \end{vmatrix} - 1\begin{vmatrix} 4 & 0 \\ 7 & -1 \end{vmatrix} + 2\begin{vmatrix} 4 & 0 \\ 7 & 3 \end{vmatrix}$$

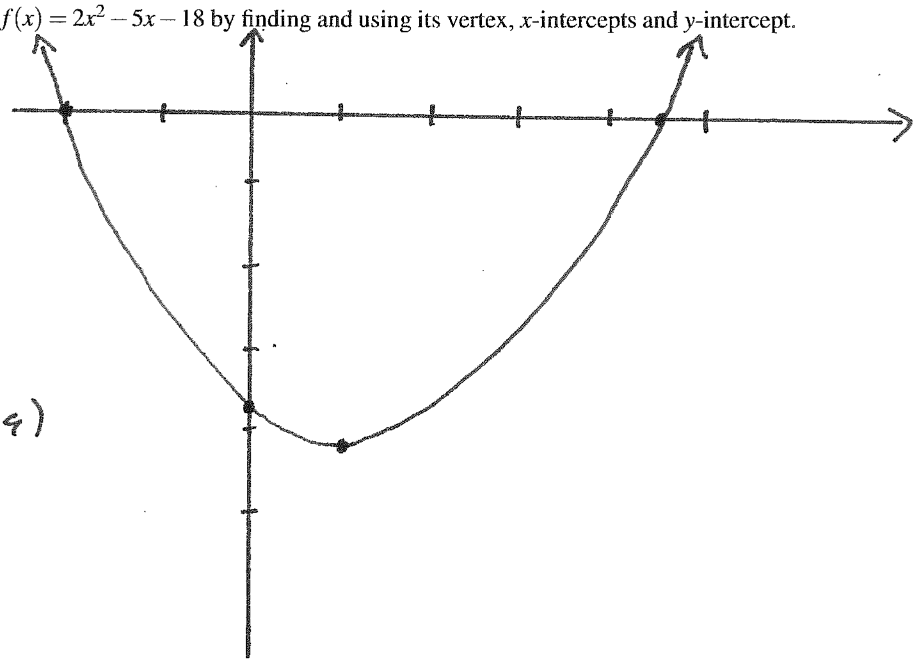
$$= 3\left[0(-1)-0(3)\right] - \left[4(-1)-0(7)\right] + 2\left[4(3)-0(7)\right]$$

$$= 4 + 24$$

$$= 28$$

$$\therefore\ y = \frac{|A_2|}{|A|} = \frac{28}{15}$$

Question 7. *(5 marks)* Sketch the graph of the function $f(x) = 2x^2 - 5x - 18$ by finding and using its vertex, x-intercepts and y-intercept.



y-int: $(0,-18)$

x-int: $0 = 2x^2 - 5x - 18$

$(2x^2)(-18) = -36x^2 = ab$

s.t. $a + b = -5x$

$-9x + 4x = -5x$

$0 = 2x^2 - 9x + 4x - 18$

$= x(2x - 9) + 2(2x - 9)$

$= (x+2)(2x-9)$

$x = -2$

$2x - 9 = 0$

$2x = 9$

$x = \frac{9}{2}$

vertex:

$$f(x) = 2x^2 - 5x - 18$$

$$= 2\left[x^2 - \frac{5}{2}x - 9\right]$$

$$= 2\left[x^2 - \frac{5}{2}x + \frac{25}{16} - \frac{25}{16} - 9\right]$$

$$= 2\left[\left(x^2 - \frac{5}{2}x + \frac{25}{16}\right) - \frac{169}{16}\right]$$

$$= 2\left[\left(x - \frac{5}{4}\right)^2 - \frac{169}{16}\right]$$

$$= 2\left(x - \frac{5}{4}\right)^2 - \frac{169}{8} \qquad \therefore \text{vertex } \left(\frac{5}{4}, -\frac{169}{8}\right)$$

Question 8. *(3 marks each)*

a. Factor $30x^4 - 8x^3 - 8x^2$

b. A right triangle with hypotenuse of length 100 has the property that its opposite side is 2 times longer than its adjacent side. Find the length of the opposite and adjacent sides.

c. A rectangle with an area of 144 has a length of one plus 3 times its width. Find the length and width of the rectangle.

a) $2x^2(15x^2-4x-4)$

$15x^2(-4) = -60x^2 = ab$

s.t. $a+b=-4x$

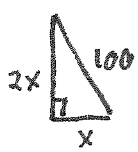
$-10x+6x=-4x$

$2x^2(15x^2-10x+6x-4)$

$= 2x^2(5x(3x-2)+2(3x-2))$

$= 2x^2(5x+2)(3x-2)$

b)



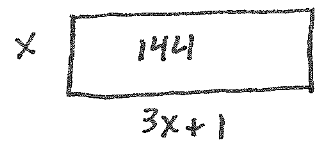
$(2x)^2+x^2=100^2$

$5x^2=10000$

$x=\pm\sqrt{2000}$

$x=\sqrt{2000}$

c)



$144 = x(3x+1)$

$144 = 3x^2+x$

$0 = 3x^2+x-144$

$x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$= \frac{-1\pm\sqrt{1^2-4(3)(144)}}{6}$

$= \frac{-1\pm\sqrt{1729}}{6}$

$\therefore x \doteq 6.76$

Question 9. *(5 marks each)* Find and verify all solutions of the system:

$3x^2 - y^2 = 11,$ (1)

$x^2 + 4y^2 = 8.$ (2)

mult (1) by 4

$12x^2-4y^2=44$ (3)

(2) + (3)

$13x^2 = 52$

$x^2 = 4$

$x = \pm\sqrt{4} = \pm 2$

Sub $x=2$ into (2)

$2^2+4y^2=8$

$y^2=1$

$y=\pm 1$

$\therefore$ (2, 1) and (2, -1) are possible solutions

Sub $x=-2$ into (2)

$(-2)^2+4y^2=8$

$y^2=1$

$y=\pm1$

$\therefore$ (-2, 1) and (-2, -1) are possible solutions

Sub (2, 1) into (1)

$3(2)^2-1^2 \stackrel{?}{=} 11$

$11 = 11$ ✓

Sub (2, -1) into (1)

$3(2)^2-(-1)^2 \stackrel{?}{=} 11$

$11 = 11$ ✓

Sub (-2, 1) into (1)

$3(-2)^2-1^2 \stackrel{?}{=} 11$

$11 = 11$ ✓

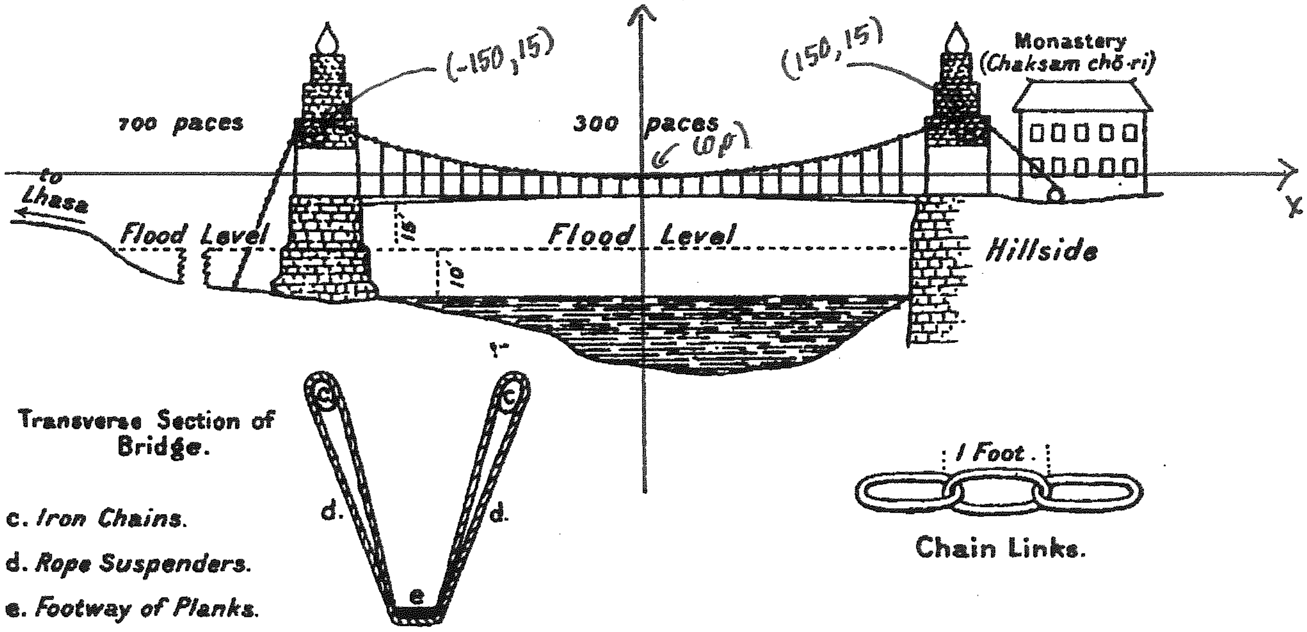
Sub (-2, -1) into (1)

$3(-2)^2-(-1)^2 \stackrel{?}{=} 11$

$11 = 11$ ✓

$\therefore$ (2, 1), (2, -1), (-2, 1) (-2, -1) are possible solutions.

Question 10. *(5 marks)* Below is a drawing of the Chakzam bridge south of Lhasa, constructed in 1430, with cables suspended between towers, and vertical suspender cables carrying the weight of a planked footway below. [1]



IRON SUSPENSION BRIDGE OVER TSANGPO.

Write the quadratic function modeling the cables suspended between the two towers. The length between the two towers is 300 paces and use the assumption that the cables at the towers is 15 paces above its lowest point. For simplicity make the origin of the function at the lowest point of the suspended cables.

Lets find a quadratic function that passes through $(-150,15)$, $(0,0)$, $(150,15)$

$f(x) = ax^2 + bx + c$

sub $(0,0)$ $0 = a(0)^2 + b(0) + c$

$0 = c$

sub $(-150,15)$

$15 = a(-150)^2 + b(-150) + c$

$15 = 22500a - 150b$ ①

sub $(150,15)$

$15 = a(150)^2 + b(150) + c$

$15 = 22500a + 150b$ ②

① + ②

$30 = 45000a$

$a = \frac{1}{1500}$

sub $a = \frac{1}{1500}$ into ①

$15 = 22500\left(\frac{1}{1500}\right) + 150b$

$15 = 15 + 150b$

$0 = b$

$\therefore f(x) = \frac{1}{1500}x^2$

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[1] A diagram of one of the earliest known suspension bridges in the world, built in 1430, at Chushul, south of Lhasa in Tibet. The image was taken by an Indian spy working for the Survey of India in 1878, and published by Waddell in 1905. – Wikipedia

Bonus Question. *(3 marks)*
From the 5th Dawson Mathematics Competition: Solve

$$x(x+y)=9,$$
$$x(y-x)=7.$$

$x^2 + xy = 9$ ①
$xy - x^2 = 7$ ②

① + ②

$2xy = 16$
$xy = 8$ ③

Sub ③ in ①

$x^2 + 8 = 9$
$x^2 = 1$
$x = \pm 1$

For $x=1$ sub into ①

$1^2 + 1 \cdot y = 9$
$y = 8$

For $x=-1$ sub into ①

$(-1)^2 + (-1)y = 9$
$y = -8$

$\therefore$ $(1,8)$ and $(-1,-8)$ are possible solutions

Verify solution $(1,8)$ by sub. into ②

$(1)(8) - {}^{2} \stackrel{?}{=} 7$
$7 = 7$

Verify solution $(-1,-8)$ by sub. into ②

$(-1)(-8) - (-1)^2 \stackrel{?}{=} 7$
$8 - 1 \stackrel{?}{=} 7$
$7 = 7$