

Name: _____
Student ID: _____

Test 3

This test is graded out of 46 marks. No books, notes, graphing calculators or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. (5 marks) Solve the following equation in the interval $[0, 2\pi]$.

$$2 \sin x \cos x - \sqrt{3} \sin x = 0$$

Question 2. (3 marks) Prove the following trigonometric identity.

$$\frac{\sec \theta}{\cos \theta} - \frac{\tan \theta}{\cot \theta} = 1$$

Question 3. (5 marks) Find the exact value of the following expression.

$\cos(255^\circ)$

Question 4. (5 marks) The compressive strength S of concrete depends, other parameters being constant, on the ratio by weight of cement to water $x = c/w$. In a certain domain of c/w , this relationship is well approximated by a straight line. Find this linear relation from these lab results. Estimate the compressive strength for a cement to water ratio of 3.5.

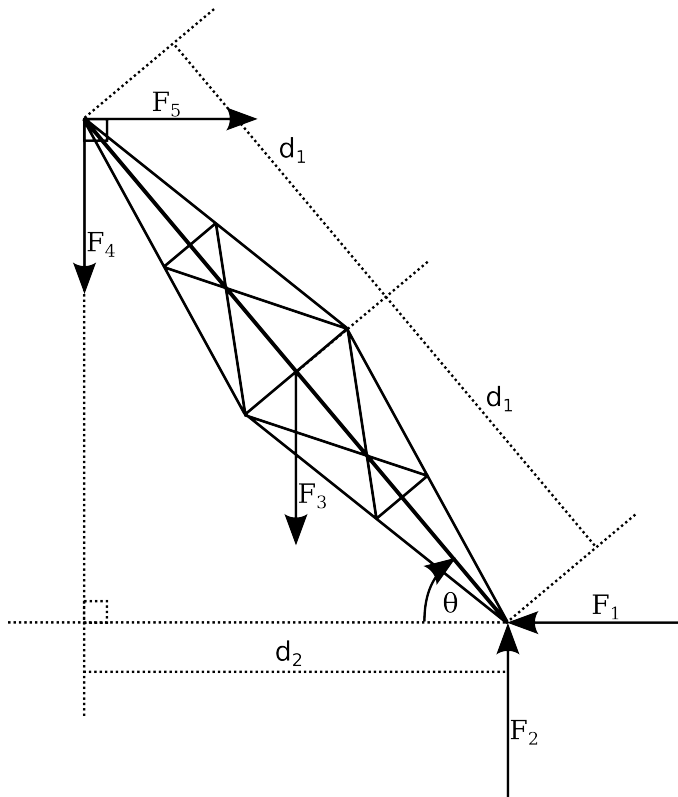
x	2.2	2.5	2.7	3.0	3.9
S	70	80	130	140	180

Question 5. (4 marks) Add the given vectors and give the resultant in terms of a magnitude and an angle in standard position.

$$A = 21.9, \quad \theta_A = 236.2^\circ$$

$$B = 96.7, \quad \theta_B = 11.5^\circ$$

Question 6. (5 marks) The following is in mechanical equilibrium. Find the missing forces, angles and/or distances, if $F_3 = 2.00$ kN, $F_4 = 15.00$ kN, $d_1 = 10.00$ m, and $d_2 = 12.00$ m.



Question 7. (5 marks) Sound pressure level (SPL) or sound level L_p is a logarithmic measure of the effective sound pressure of a sound relative to a reference value. It is measured in decibels (dB) above a standard reference level.

$$L_p = 10 \log_{10} \left(\frac{p_{\text{rms}}^2}{p_{\text{ref}}^2} \right) \text{ dB}$$

where $p_{\text{ref}} = 20 \mu\text{P}$ is the reference sound pressure and p_{rms} is the rms sound pressure being measured.¹

According to the article *Traffic Induced Noise Pollution in Dhaka City* “the average noise level in the road side in Dhaka city is about 78dB(A) which far exceeds the acceptable limit of 60 dB(A) set by the Department of Environment”.

Assuming dB = dB(A) in the above equation find p_{rms} when the sound pressure level is measured at 78dB(A).

Question 8. (4 marks) Rewrite the expression

$$2 \log x - 5 \log(x^2 + 1) + 4 \log(x - 1)$$

as a single logarithm $\log f(x)$. Find $f(x)$.

¹Wikipedia.org: Sound pressure

Question 9. (5 marks) Find the point(s) of intersection of the line passing through the point $(4, 3)$ and perpendicular to the line $5y + 4x = -15$ and the circle $(x - 1)^2 + (y + 1)^2 = 25$. Decide whether this line is a tangent line, secant line, or neither.

Question 10. In curing concrete the strength after t days of curing is given by the equation

$$f = f_c(1 - e^{-kt})$$

where f_c is the ultimate strength and is given to be 50 MP.

- a. (3 marks) Solve for k .
- b. (1 mark) If the strength of the concrete is 15 MP after 5 days, find k .
- c. (1 mark) Determine the strength of the concrete after 30 days.

Bonus Question. (5 marks) From the 5th Dawson Mathematics Competition: Solve

$$(x+y)^x = (x-y)^y$$

$$\log_2 x - \log_2 y = 1.$$

Formulae:

Equation of the least squares line: $y = mx + b$

$$m = \frac{n \sum xy - (\sum x)(\sum y)}{n \sum x^2 - (\sum x)^2}$$

$$b = \frac{(\sum x^2)(\sum y) - (\sum xy)(\sum x)}{n \sum x^2 - (\sum x)^2}$$

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

$$\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$$

$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

$$\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}$$

$$\tan(\alpha - \beta) = \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \tan \beta}$$

$$\sin 2\alpha = 2 \sin \alpha \cos \alpha$$

$$\cos 2\alpha = \cos^2 \alpha - \sin^2 \alpha$$

$$\cos 2\alpha = 2 \cos^2 \alpha - 1$$

$$\cos 2\alpha = 1 - 2 \sin^2 \alpha$$

$$\tan 2\alpha = \frac{2 \tan \alpha}{1 - \tan^2 \alpha}$$