Dawson College: Introduction to Applied Mathematics: 201-912-DW-S1: Fall 2011

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Test 3

This test is graded out of 48 marks. No books, notes, graphing calculators or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. *(5 marks)* Solve the following equation in the interval $[0, 2\pi]$.

$2\sin x\cos x - \sqrt{3}\sin x = 0$

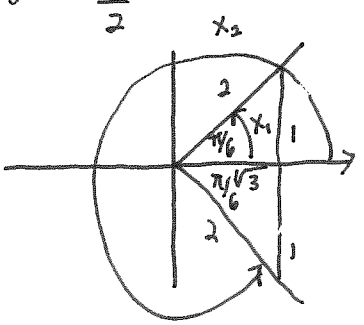
$\sin x(2\cos x - \sqrt{3}) = 0$

$\sin x = 0$ | $2\cos x - \sqrt{3} = 0$

$2\cos x = \sqrt{3}$

$\cos x = \frac{\sqrt{3}}{2}$

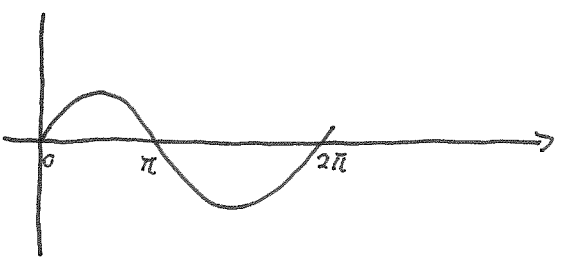
For $\cos x = \frac{\sqrt{3}}{2}$



$x_1 = \frac{\pi}{6}$

$x_2 = \frac{11\pi}{6}$

For $\sin x = 0$



$x_3 = 0$

$x_4 = \pi$

$x_5 = 2\pi$

Question 2. *(3 marks)* Prove the following trigonometric identity.

$$\frac{\sec\theta}{\cos\theta} - \frac{\tan\theta}{\cot\theta} = 1$$

$$LHS = \frac{\frac{1}{\cos\theta}}{\cos\theta} - \frac{\frac{\sin\theta}{\cos\theta}}{\frac{\cos\theta}{\sin\theta}}$$

$$= \frac{1}{\cos^2\theta} - \frac{\sin\theta}{\cos\theta}\cdot\frac{\sin\theta}{\cos\theta}$$

$$= \frac{1}{\cos^2\theta} - \frac{\sin^2\theta}{\cos^2\theta}$$

$$= \frac{1-\sin^2\theta}{\cos^2\theta} = \frac{\cos^2\theta}{\cos^2\theta} = 1 = RHS.$$

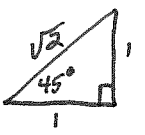
Question 3. *(5 marks)* Find the exact value of the following expression.

$\cos(255°) = \cos(210° + 45°)$

$= \cos(210°)\cos(45°) - \sin(210°)\sin(45°)$

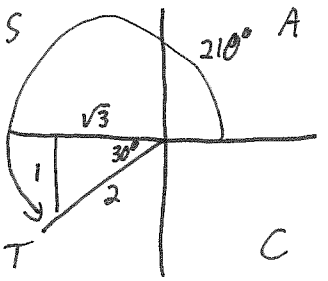
$= -\frac{\sqrt{3}}{2} \cdot \frac{1}{\sqrt{2}} - \left(-\frac{1}{2}\right) \cdot \frac{1}{\sqrt{2}}$

$= \frac{1 - \sqrt{3}}{2\sqrt{2}}$



$\cos 45° = \frac{1}{\sqrt{2}}$

$\sin 45° = \frac{1}{\sqrt{2}}$



$\cos(210°) = -\frac{\text{adj}}{\text{hyp}} = -\frac{\sqrt{3}}{2}$

$\sin(210°) = -\frac{\text{opp}}{\text{hyp}} = -\frac{1}{2}$

Question 4. *(5 marks)* The compressive strength S of concrete depends, other parameters being constant, on the ratio by weight of cement to water $x = c/w$. In a certain domain of c/w, this relationship is well approximated by a straight line. Find this linear relation from these lab results. Estimate the compressive strength for a cement to water ratio of 3.5.

x	2.2	2.5	2.7	3.0	3.9
S	70	80	130	140	180

Let $y = S$ and $n = 5$

x	y	xy	x^2
2.2	70	154	4.84
2.5	80	200	6.25
2.7	130	351	7.29
3.0	140	420	9
3.9	180	702	15.21
14.3	600	1827	42.59

$$m = \frac{n\sum xy - (\sum x)(\sum y)}{n\sum x^2 - (\sum x)^2}$$

$$= \frac{5(1827) - (14.3)(600)}{5(42.59) - (14.3)^2}$$

$$= 65.60$$

$$b = \frac{(\sum x^2)(\sum y) - (\sum xy)(\sum x)}{n\sum x^2 - (\sum x)^2}$$

$$= \frac{(42.59)(600) - (1827)(14.3)}{5(42.59) - (14.3)^2}$$

$$= -67.62$$

$\therefore y = 65.60x - 67.62$

$\therefore$ when the cement to water ratio is 3.5. the strength is estimated to be $65.60(3.5) - 67.62$

$= 162$

Question 5. *(4 marks)* Add the given vectors and give the resultant in terms of a magnitude and an angle in standard position.

$A = 21.9, \quad \theta_A = 236.2°$

$B = 96.7, \quad \theta_B = 11.5°$

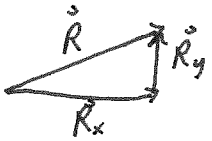
$(A_x, A_y) = (21.9\cos 236.2°,\ 21.9\sin 236.2°) = (-12.2,\ -18.2)$

$(B_x, B_y) = (96.7\cos 11.5°,\ 96.7\sin 11.5°) = (94.8,\ 19.3)$

$\therefore \vec{A} + \vec{B} = (-12.2, -18.2) + (94.8,\ 19.3)$

$= (82.6,\ 1.1) = \vec{R}$

So



$R = \sqrt{82.6^2 + 1.1^2}$

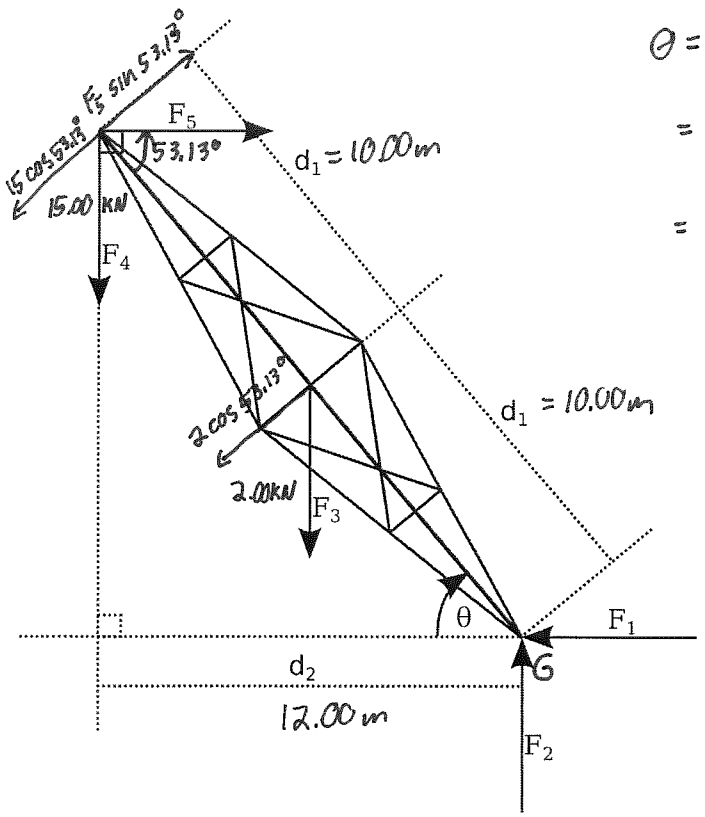
$= 82.6$

$\theta = \arctan\left(\frac{R_y}{R_x}\right)$

$= \arctan\left(\frac{1.1}{82.6}\right)$

$= 0.8°$

Question 6. *(5 marks)* The following is in mechanical equilibrium. Find the missing forces, angles and/or distances, if $F_3 = 2.00$ kN, $F_4 = 15.00$ kN, $d_1 = 10.00$ m, and $d_2 = 12.00$ m.



$\theta = \arccos\left(\frac{d_2}{d_1 + d_1}\right)$

$= \arccos\left(\frac{12.00}{20.00}\right)$

$= 53.13°$

For mechanical equilibrium

(i) $\sum F_i = 0$

(ii) $\sum \Gamma_i = 0$

To satisfy (i) $\sum F_{xi} = 0$ and $\sum F_{yi} = 0$

$F_5 - F_1 = 0$ $\qquad F_2 - F_4 - F_3 = 0$

$F_5 = F_1$ $\qquad F_2 = F_4 + F_3$

$F_2 = 17.00$ kN

To satisfy (ii)

cw moment about G = ccw moment about G

$F_5 \sin 53.13° = 15\cos 53.15° + 2\cos 53.13°$

$F_5 = \frac{15\cos 53.15° + 2\cos 53.13°}{\sin 53.15°}$

$= 12.74$ kN

$\therefore F_1 = 12.74$ kN

Question 7. *(5 marks)* Sound pressure level (SPL) or sound level L_p is a logarithmic measure of the effective sound pressure of a sound relative to a reference value. It is measured in decibels (dB) above a standard reference level.

$$L_p = 10\log_{10}\left(\frac{{p_{\text{rms}}}^2}{{p_{\text{ref}}}^2}\right) \text{ dB}$$

where $p_{\text{ref}} = 20\mu\text{P}$ is the reference sound pressure and p_{rms} is the rms sound pressure being measured.[1]
According to the article *Traffic Induced Noise Pollution in Dhaka City* "the average noise level in the road side in Dhaka city is about 78dB(A) which far exceeds the acceptable limit of 60 dB(A) set by the Department of Environment".

Assuming dB = dB(A) in the above equation find p_{rms} when the sound pressure level is measured at 78dB(A).

$$78 = 10\log_{10}\left(\frac{P_{rms}^2}{P_{ref}^2}\right)$$

$$7.8 = \log_{10}\left(\frac{P_{rms}^2}{20^2}\right)$$

$$10^{7.8} = 10^{\log_{10}\left(\frac{P_{rms}^2}{20^2}\right)}$$

$$\frac{P_{rms}^2}{20^2} = 10^{7.8}$$

$$P_{rms}^2 = 20^2 10^{7.8}$$

$$P_{rms} = \sqrt{20^2 10^{7.8}}$$

$$= 20 \cdot 10^{3.9}$$

$$= 158866$$

$$= 160000\mu P$$

Question 8. *(4 marks)* Rewrite the expression

$$2\log x - 5\log(x^2+1) + 4\log(x-1) = \log x^2 - \log(x^2+1)^5 + \log(x-1)^4$$

as a single logarithm $\log f(x)$. Find $f(x)$.

$$= \log \frac{x^2(x-1)^4}{(x^2+1)^5}$$

$$= \log f(x) \quad \text{where} \quad f(x) = \frac{x^2(x-1)^4}{(x^2+1)^5}$$

[1] Wikipedia.org: Sound pressure

Question 9. *(5 marks)* Find the point(s) of intersection of the line passing through the point $(4,3)$ and perpendicular to the line $5y+4x=-15$ and the circle $(x-1)^2+(y+1)^2=25$. Decide whether this line is a tangent line, secant line, or neither.

$y = \frac{-4}{5}x - \frac{15}{5}$ $\therefore$ $\perp$ line will have slope $m_\perp = \frac{5}{4}$

So $y = m_\perp x + b$ and passes through $(4,3)$

$3 = \frac{5}{4}(4) + b$

$-2 = b$

$\therefore y = \frac{5}{4}x - 2$

Sub above in eqn. of circle

$(x-1)^2 + \left(\frac{5}{4}x - 2 + 1\right)^2 = 25$

$(x-1)^2 + \left(\frac{5}{4}x - 1\right)^2 = 25$

$x^2 - 2x + 1 + \frac{25}{16}x^2 - \frac{5}{2}x + 1 = 25$

$16x^2 - 32x + 16 + 25x^2 - 40x + 16 = 25(16)$

$41x^2 - 72x + 32 = 400$

$41x^2 - 72x - 368 = 0$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{72 \pm \sqrt{(-72)^2 - 4(41)(-368)}}{2(41)}$

$= \frac{72 \pm \sqrt{65536}}{82}$

$= \frac{72 \pm 256}{82}$

$= 4$ and $\frac{-92}{41}$

Let $x = 4$

$y = \frac{5}{4}(4) - 2 = 3$

$\therefore (4,3)$

Let $x = -\frac{92}{41}$

$y = \frac{5}{4}\left(-\frac{92}{41}\right) - 2$

$= \frac{-460}{164} - 2$

$= \frac{-197}{41}$

verify sol.

$(4-1)^2 + (3+1)^2 \stackrel{?}{=} 25$

$3^2 + 4^2 \stackrel{?}{=} 25$

$25 = 25$ ✓

$\left(\frac{-92}{41} - 1\right)^2 + \left(\frac{-197}{41} + 1\right)^2 \stackrel{?}{=} 25$

$\left(\frac{-133}{41}\right)^2 + \left(\frac{-156}{41}\right)^2 \stackrel{?}{=} 25$

$\frac{17689}{41^2} + \frac{24336}{41^2} \stackrel{?}{=} 25$

$25 = 25$ ✓

$\therefore$ the line is secant to the circle.

Question 10. *(5 marks)* In curing concrete the strength after t days of curing is given by the equation

$$f = f_c(1 - e^{-kt})$$

where f_c is the ultimate strength and is given to be 50 MP.

a. *(3 marks)* Solve for k.

b. *(1 mark)* If the strength of the concrete is 15 MP after 5 days, find k.

c. *(1 mark)* Determine the strength of the concrete after 30 days.

a) $f = f_c - f_c e^{-Kt}$

$f_c e^{-Kt} = f_c - f$

$e^{-Kt} = \frac{f_c - f}{f_c}$

$\ln e^{-Kt} = \ln\left(\frac{f_c - f}{f_c}\right)$

$-Kt = \ln\left(\frac{f_c - f}{f_c}\right)$

$K = \frac{\ln\left(\frac{f_c - f}{f_c}\right)}{-t}$

b) $K = \frac{\ln\left(\frac{50-15}{50}\right)}{-5}$

$= \frac{\ln\left(\frac{35}{50}\right)}{-5}$

$= 0.07133$

c) $f = 50(1 - e^{-0.07133(30)})$

$= 44\ MP$

Bonus Question. *(5 marks)* From the 5th Dawson Mathematics Competition: Solve

$(x+y)^x = (x-y)^y$ ①
$\log_2 x - \log_2 y = 1.$ ②

$\log_2 \frac{x}{y} = 1$

$2^{\log_2 \frac{x}{y}} = 2^1$

$\frac{x}{y} = 2$

$x = 2y$ ③

Sub ③ into ①

$(2y+y)^{2y} = (2y-y)^y$

$(3y)^{2y} = (y)^y$

$\ln(3y)^{2y} = \ln y^y$

$0 = \ln(3y)^{2y} - \ln y^y$

$0 = \ln\left(\frac{(3y)^{2y}}{y^y}\right)$

$0 = \ln\left(\frac{(3y)^2}{y}\right)^y$

$0 = y \ln(9y)$

$y \neq 0$ not valid solution

$\ln(9y) = 0$

$e^{\ln(9y)} = e^0$

$9y = 1$

$y = \frac{1}{9}$

$\therefore x = 2y = 2\left(\frac{1}{9}\right) = \frac{2}{9}$

Verify solution

$\left(\frac{2}{9}+\frac{1}{9}\right)^{2/9} \stackrel{?}{=} \left(\frac{2}{9}-\frac{1}{9}\right)^{1/9}$

$\left(\frac{3}{9}\right)^{2/9} \stackrel{?}{=} \sqrt[9]{1/9}$

$\left(\frac{1}{3}\right)^{2/9} \stackrel{?}{=} \sqrt[9]{1/9}$

$\sqrt[9]{1/9} = \sqrt[9]{1/9}$ ✓

$\log_2\left(\frac{2}{9}\right) - \log_2\left(\frac{1}{9}\right) \stackrel{?}{=} 1$

$\log_2\left(\frac{2/9}{1/9}\right) \stackrel{?}{=} 1$

$\log_2 2 \stackrel{?}{=} 1$

$1 = 1$ ✓