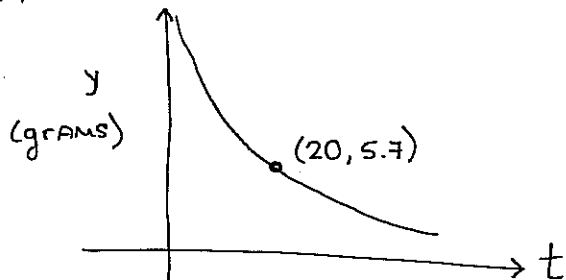


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BONUS EXERCISES  
 LOGARITHMS & EXPONENTIAL  
 FUNCTIONS  
 943-DW APPLIED MATH  
 FALL 2011

(1) THE HALF-LIFE FOR THE ELEMENT WITH THE GRAPH OF DECAY BELOW IS 15 YEARS.



FIND THE INITIAL AMOUNT OF THE ELEMENT

soln.

$$P = P_0 e^{kt}$$

$$P = \frac{1}{2} P_0 \quad \text{WHEN } t = 15$$

$$\frac{1}{2} P_0 = P_0 e^{15k}$$

$$\frac{1}{2} = e^{15k} \rightarrow \ln \frac{1}{2} = \ln e^{15k}$$

$$\ln \frac{1}{2} = 15k$$

$$k = \frac{\ln \frac{1}{2}}{15} \approx -0.0462$$

$$P = P_0 e^{-0.0462t}$$

$$P = P_0 e^{-0.0462t}$$

$$5.7 = P_0 e^{-0.0462(20)}$$

$$P_0 = \frac{5.7}{e^{-0.0462(20)}} \approx 14.4 \text{ grams}$$

SOLVE FOR  $x$ :

(2)  $\log x + \log(x-15) = 2$

$$\log(x(x-15)) = 2$$

$$10^2 = x^2 - 15x$$

$$x^2 - 15x + 100 = 0 \rightarrow$$

$$(x-20)(x+5) = 0$$

$$x = 20 \text{ \& } x = -5$$

ONLY  $x = 20$  IS A SOLUTION!

$$\textcircled{3} \quad \log(95x+10) - \log(x-1) = 2$$

$$\log\left(\frac{95x+10}{x-1}\right) = 2$$

$$10^2 = \frac{95x+10}{x-1}$$

$$100(x-1) = 95x+10$$

$$100x - 100 = 95x + 10$$

$$5x = 110$$

$$x = \frac{110}{5} = 22$$

$$\textcircled{4} \quad \log_6(x+3) = 1 - \log_6(x+4)$$

$$\log_6(x+3) + \log_6(x+4) = 1$$

$$\log_6(x+3)(x+4) = 1$$

$$6 = (x+3)(x+4)$$

$$x^2 + 3x + 4x + 12 = 6$$

$$x^2 + 7x + 12 = 6$$

$$x^2 + 7x + 6 = 0$$

$$(x+6)(x+1) = 0$$

$$x = -6 \text{ \& } x = -1$$

ONLY  $x = -1$   
IS A SOLUTION

$$\textcircled{5} \quad \ln(x+1) - \ln(x-1) - 1 = 0$$

$$\ln\left(\frac{x+1}{x-1}\right) = 1$$

$$e^1 = \frac{x+1}{x-1}$$

$$e(x-1) = x+1$$

$$ex - e = x + 1$$

$$ex - x = e + 1$$

$$x(e-1) = e+1$$

$$x = \frac{e+1}{e-1}$$

$$\ln(x-2) + \ln(2x+1) = 2 \ln x$$

$$\ln[(x-2)(2x+1)] = \ln x^2$$

$$(x-2)(2x+1) = x^2$$

$$2x^2 - 4x + x - 2 = x^2$$

$$x^2 - 3x - 2 = 0$$

$$x = \frac{3 \pm \sqrt{9 - 4(1)(-2)}}{2}$$

$$= \frac{3 \pm \sqrt{17}}{2}$$

ONLY  $x = \frac{3 + \sqrt{17}}{2}$  IS A SOLUTION

$$\textcircled{7} \quad e^{-x} = 3.82$$

$$\ln e^{-x} = \ln 3.82$$

$$-x = \ln 3.82$$

$$x = -\ln 3.82$$

$$\textcircled{8} \quad 3^{2x+1} = 457$$

$$\ln 3^{2x+1} = \ln 457$$

$$(2x+1)\ln 3 = \ln 457$$

$$2x+1 = \frac{\ln 457}{\ln 3}$$

$$2x = \frac{\ln 457}{\ln 3} - 1$$

$$x = \frac{\ln 457}{2 \ln 3} - \frac{1}{2}$$

$$\textcircled{9} \quad 7^{x^2} = 10$$

$$\ln 7^{x^2} = \ln 10$$

$$x^2 \ln 7 = \ln 10$$

$$x^2 = \frac{\ln 10}{\ln 7}$$

$$x = \pm \sqrt{\frac{\ln 10}{\ln 7}}$$

$$\textcircled{10} \quad 2^x = 5^{x-1}$$

$$\ln 2^x = \ln 5^{x-1}$$

$$x \ln 2 = (x-1) \ln 5$$

$$x \ln 2 = x \ln 5 - \ln 5$$

$$x \ln 2 - x \ln 5 = -\ln 5$$

$$x (\ln 2 - \ln 5) = -\ln 5$$

$$x = \frac{-\ln 5}{\ln 2 - \ln 5}$$

(11)

$$3(5^{x+1}) = 4^{1-x}$$

$$\ln(3(5^{x+1})) = \ln(4^{1-x})$$

$$\ln 3 + \ln 5^{x+1} = \ln 4^{1-x}$$

$$\ln 3 + (x+1)\ln 5 = (1-x)\ln 4$$

$$\ln 3 + x\ln 5 + \ln 5 = \ln 4 - x\ln 4$$

$$x\ln 5 + x\ln 4 = \ln 4 - \ln 5 - \ln 3$$

$$x(\ln 5 + \ln 4) = \ln 4 - \ln 5 - \ln 3$$

$$x = \frac{\ln 4 - \ln 5 - \ln 3}{\ln 5 + \ln 4} = \boxed{\frac{\ln\left(\frac{4}{15}\right)}{\ln(20)}}$$

(12)

$$e^{2x+1} \cdot e^{-4x} = 3e$$

$$e^{-2x+1} = 3e$$

$$\ln e^{-2x+1} = \ln 3e$$

$$-2x+1 = \ln 3 + \ln e$$

$$-2x = \ln 3$$

$$x = \boxed{\frac{-\ln 3}{2}}$$

EVALUATE WITH YOUR CALCULATOR

$$(13) e^{-\log_9 99}$$

$$= e^{-\frac{\log 99}{\log 9}}$$

$$= e^{\frac{-1.99}{0.95}}$$

$$= e^{-2.1} \approx \boxed{0.12}$$

$$(14) (\log_5 7)^{\frac{1}{3}}$$

$$= \left(\frac{\log 7}{\log 5}\right)^{\frac{1}{3}}$$

$$= \boxed{1.06}$$