FINAL EXAMINATION - WINTER 2010

Department of Mathematics, Dawson College

2:00 - 5:00pm, Dec. 17th 2010

201-943-DW: Applied Mathematics for Electronics Engineering

Examiners: Parviz Haggi-Mani and Emilie Richer

Name: SOLUTIONS

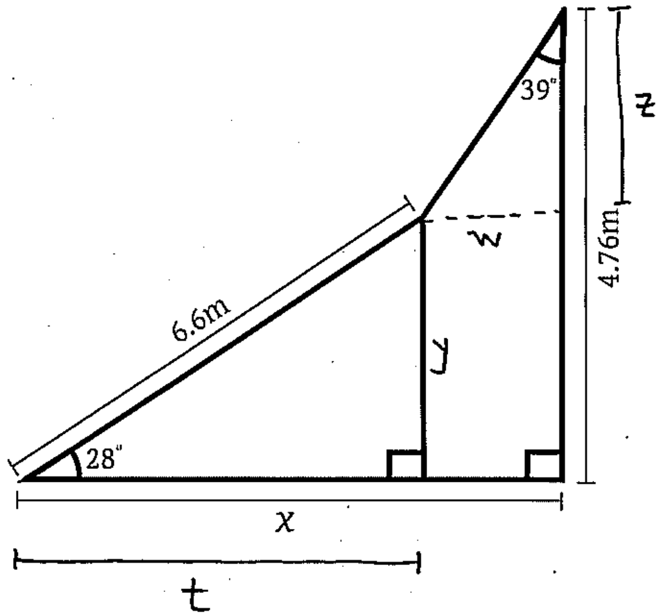
Student ID: ____________________

Instructions:

- Print your name and student ID number in the space provided above.
- All questions are to be answered directly on the examination paper in the space provided. If you need more space for your answer use the back of the page.
- No books, notes, graphing calculators, programmable calculators or cell phones are permitted.
- SHOW ALL YOUR WORK. Show all your work clearly and justify all your answers.
- Verify that your final examination copy has a total of 13 pages including this cover page.

Question	# Marks	
1	5	
2	6	
3	8	
4	8	
5	8	
6	4	
7	6	
8	4	
9	6	
10	8	
11	4	
12	4	
13	4	
14	3	
15	5	
16	8	
17	5	
18	4	
Total	100	

Question 1. *(5 marks)*
Find the length x in the diagram given below.



$$\sin 28^\circ = \frac{y}{6.6} \Rightarrow y = 6.6 \sin 28^\circ$$
$$= 3.098\,m$$

$$z = 4.76 - 3.098 = 1.66\,m$$

$$\tan 39^\circ = \frac{w}{1.66} \Longrightarrow w = 1.66 \tan 39^\circ$$
$$= 1.344\,m$$

$$\cos 28^\circ = \frac{t}{6.6} \Longrightarrow t = 5.83\,m$$

FINALLY $x = t + w$
$$= 5.83 + 1.344 = \boxed{7.17\,m}$$

Question 2. *(6 marks)*
Solve the following equations.

a. $\log_8 x = -2$

$x = 8^{-2} = 1/64$ $\boxed{x = 1/64}$

b. $\log_b(\frac{1}{16}) = -3$

$b^{-3} = \frac{1}{16}$ $b^3 = 16$ $\boxed{b = \sqrt[3]{16}}$

c. $\log_5 125^{-1} = x+1$

$\log_5 5^{-3} = x+1$
$-3 = x+1$ $\boxed{x=-4}$

Question 3. *(8 marks)*
Factor the given expressions completely.

a. $x^2 - 4x - 45$ $= (x-9)(x+5)$

b. $2k^2 - k - 36$ $= (k+4)(2k-9)$

c. $4x^2 - 64y^4$ $= 4(x-4y^2)(x+4y^2)$

d. $16x^3y + 54y$ $= 2y(8y^3+27)$
$= 2y(2x+3)(4x^2-6x+9)$

Question 4.

Solve each system of equations.

a. *(2 marks)*

$2x - 3y = -5$
$3x + 2y = 12$

$$\boxed{\begin{aligned} y &= 3 \\ x &= 2 \end{aligned}}$$

b. *(4 marks)*

$3r + s - t = 2$
$r - 2s + t = 0$
$4r - s + t = 3$

$$\boxed{\begin{aligned} r &= 5/7 \\ s &= 6/7 \\ t &= 1 \end{aligned}}$$

Question 5.
Given the function $f(x) = 2 + 3x + x^2$
a. *(5 marks)*
Graph the function $y = f(x)$ indicating its vertex x-intercepts and y-intercept.

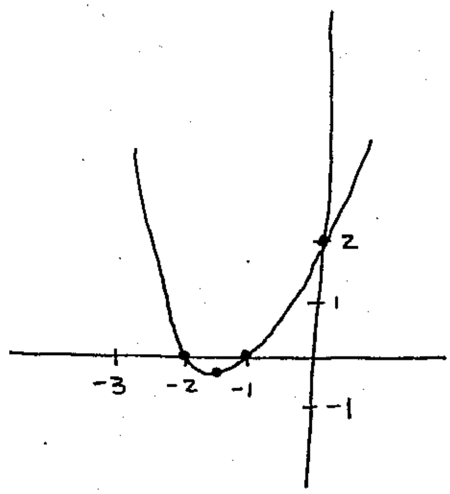
Y-INTERCEPT $(0,2)$
X-INTERCEPTS : $(-1,0)$ & $(-2,0)$

$$0 = x^2 + 3x + 2$$
$$= (x+2)(x+1)$$
$$x = -2, -1$$

VERTEX : $x = \frac{-b}{2a} = \frac{-3}{2}$

$$Y = 2 + 3\left(-\tfrac{3}{2}\right) + \left(-\tfrac{3}{2}\right)^2$$
$$= 2 - \tfrac{9}{2} + \tfrac{9}{4}$$
$$= -\tfrac{1}{4}$$

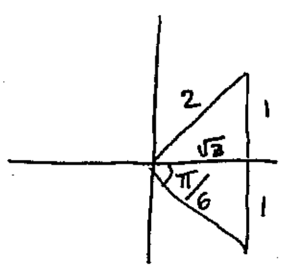
VERTEX : $\left(-\tfrac{3}{2}, -\tfrac{1}{4}\right)$



b. *(3 marks)*
State the domain and range of f.

DOMAIN $\mathbb{R}$
RANGE $[-\tfrac{1}{4}, \infty)$

Question 6. *(4 marks)*
Given $\cos\theta = \frac{\sqrt{3}}{2}$, find θ for $0 \le \theta < 2\pi$



$$\boxed{\theta = \frac{\pi}{6}, \frac{11\pi}{6}}$$

Question 7. *(4 marks)*
a. Find the slope and y-intercept of the line $3x - 7y = 6$

SLOPE $3/7$

Y-INTERCEPT $-6/7$

b. Find the value of k such that the line $kx - 2y = 9$ has a slope of 3.

$\boxed{K = 6}$

$-2Y = -Kx + 9$

$Y = K/2\, x - 9/2$

$3 = K/2$

$K = 6$

Question 8. *(4 marks)*
Express as the logarithm of a single quality.

a. $2\log_5 x - 3\log_5(x+1)$

$$\boxed{\log_5\left(\frac{x^2}{(x+1)^3}\right)}$$

b. $\frac{7\ln x}{\ln 2} - \ln(\frac{x}{3})$

$$\boxed{\ln\left(\frac{3x^{7/\ln 2}}{x}\right)}$$

Question 9. *(8 marks)*
Simplify the given expressions by performing the indicated operations and expressing all answers in the form $a+bj$.

a. $j^{18}-j^{73}-j^{311}$

$= \boxed{-1-2j}$

b. $(-2.3j)(1.2+5j)$

$\boxed{11.5-2.76j}$

c. $j\sqrt{-9}-j^5\sqrt{-16}+3j^2$

$\boxed{-2}$

d. $\frac{1-2j}{3+4j}$

$\boxed{\frac{11}{25}-\frac{10}{25}j}$

Question 10.
Solve the following equations.
a. *(2 marks)*
$2^{x+1} = 0.75$

$$x = \frac{\ln 0.75}{\ln 2} - 1$$

$$\boxed{x = -1.415}$$

b. *(3 marks)*
$\log_3(x-2) + \log_3(x) = 1$

$$\log_3[(x-2)(x)] = 1$$
$$x^2 - 2x = 3$$
$$x^2 - 2x - 3 = 0$$
$$(x-3)(x+1) = 0$$
$$x = 3 \quad x = -1$$

$$\boxed{x = 3}$$

c. *(3 marks)*
$2(5^x) = 3^{x+1}$

$$x = \frac{\ln 3 - \ln 2}{\ln 5 - \ln 3}$$

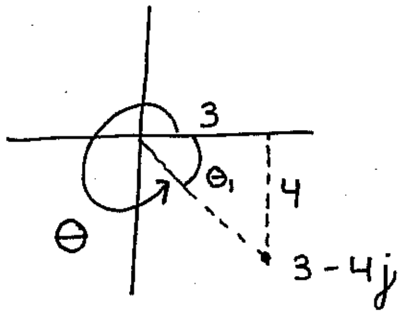
$$\boxed{x = 0.79}$$

Question 11. *(3 marks)*
If $\log_b x = 2$ and $\log_b y = 3$ then find the value of $\log_b \sqrt{x^5 y^3}$

$$\log_b \sqrt{x^5 y^3} = \frac{1}{2} \log_b x^5 y^3$$
$$= \frac{1}{2} [5 \log_b x + 3 \log_b y]$$
$$= \frac{1}{2} [5(2) + 3(3)] = \boxed{9.5}$$

Question 12. *(6 marks)*
Perform the indicated operation. Express the result in rectangular, exponential and polar forms.

$(3-4j)^6$



$$r^2 = 3^2 + 4^2$$
$$r = 5$$

$$\theta_1 = \tan^{-1}\left(\tfrac{4}{3}\right)$$
$$= 53.13^\circ$$

$$\theta = 360^\circ - 53.13^\circ$$
$$= 306.87^\circ$$

$$3-4j = 5e^{306.87j}$$

so $(3-4j)^6 = 5^6\left(e^{306.87j}\right)^6$

(IN EXPO.) $= 5^6 e^{1841.22j}$

(IN POLAR)

$$(3-4j)^6 = 5^6\left(\cos 1841.22^\circ + \sin 1841.22^\circ j\right)$$

(IN RECTANGULAR) $= \boxed{11752.89 + 10296.13j}$

Question 13. *(8 marks)*
Find θ for $0° \leq \theta < 360°$

a. If $\csc\theta = -7.62$

$$\boxed{\theta = 187.54°,\ 352.46°}$$

b. If $\tan\theta = 1.35$ and $\sin\theta < 0$

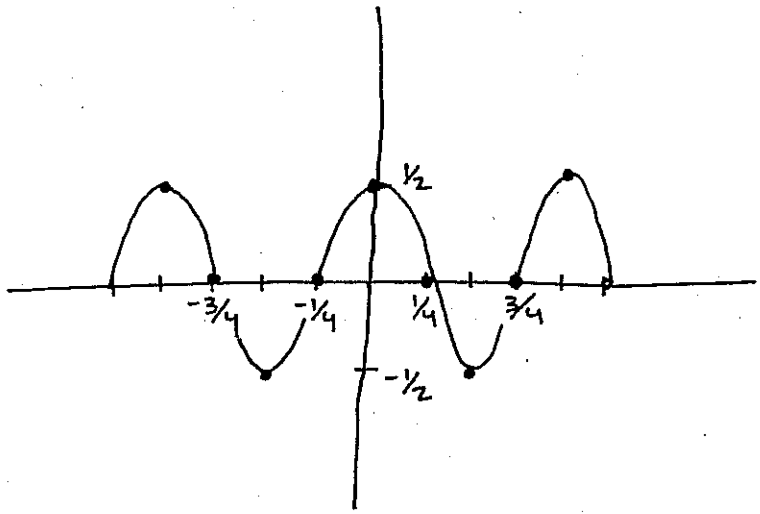
$$\boxed{\theta = 269.58°}$$

Question 14. *(3 marks)*
Isolate the variable μ in the equation $I = \frac{VR_2+VR_1(1+\mu)}{R_1R_2}$.

$$\boxed{\mu = \frac{IR_1R_2 - VR_2 - VR_1}{VR_1}}$$

Question 15. *(5 marks)*
Graph the function $y = \frac{1}{2}\cos(2\pi x)$ over two periods. State its period and its amplitude.

x	$\theta = 2\pi x$	y
-1	-2π	$\frac{1}{2}$
$-\frac{3}{4}$	$-\frac{3\pi}{2}$	0
$-\frac{1}{2}$	$-\pi$	$-\frac{1}{2}$
$-\frac{1}{4}$	$-\frac{\pi}{2}$	0
0	0	$\frac{1}{2}$
$\frac{1}{4}$	$\frac{\pi}{2}$	0
$\frac{1}{2}$	π	$-\frac{1}{2}$
$\frac{3}{4}$	$\frac{3\pi}{2}$	0
1	2π	$\frac{1}{2}$



Question 16.
Solve the following equations.
a. *(3 marks)*
$6x^2 = 9 - 4x$

$$6x^2 + 4x - 9 = 0$$

$$x = \frac{-4 \pm \sqrt{16 - 4(6)(-9)}}{2(6)} = \frac{-4 \pm \sqrt{232}}{12}$$

b. $\frac{x-2}{x-5} = \frac{15}{x^2-5x}$ *(5 marks)*

$$\frac{x-2}{x-5} = \frac{15}{x(x-5)}$$

$$x(x-2) = 15$$
$$x^2 - 2x - 15 = 0$$
$$(x-5)(x+3) = 0$$

$$x = 5, -3$$

ONLY $\boxed{x = -3}$ is a solution

Question 17 *(5 marks)*
Find the cube roots of -64.

SOLUTION 1

$$x^3 = -64$$
$$x^3 + 64 = 0$$
$$(x+4)(x^2-4x+16) = 0$$

$$x = -4$$

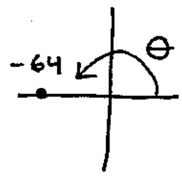
$$x = \frac{4 \pm \sqrt{16-4(16)}}{2}$$
$$= \frac{4 \pm 4\sqrt{3}\,j}{2}$$
$$= 2 \pm 2\sqrt{3}\,j$$

Solutions

$$\boxed{-4,\ 2+2\sqrt{3}\,j,\ 2-2\sqrt{3}\,j}$$

SOLUTION 2

$$x = (-64)^{1/3}$$



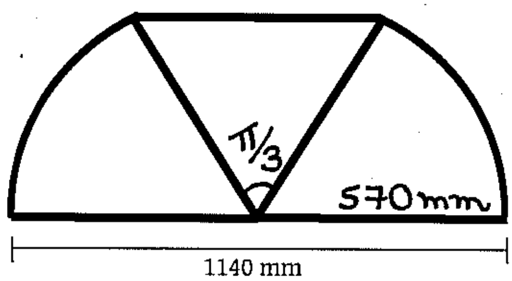
$$\theta = 180^\circ,\ 540^\circ,\ 900^\circ$$
$$r = 64$$

① $$(-64)^{1/3} = \left(64e^{180^\circ j}\right)^{1/3}$$
$$= 4e^{60^\circ j}$$
$$= 4\cos 60^\circ + 4\sin 60^\circ j$$
$$= \boxed{2 + 2\sqrt{3}\,j}$$

② $$(-64)^{1/3} = \left(64e^{540^\circ j}\right)^{1/3}$$
$$= 4e^{180^\circ j}$$
$$= 4\cos 180^\circ \cancel{j} + 4\sin 180^\circ j$$
$$= \boxed{-4}$$

③ $$(-64)^{1/3} = \left(64e^{900^\circ j}\right)^{1/3}$$
$$= 4\cos 300^\circ + 4\sin 300^\circ j$$
$$= \boxed{2 - 2\sqrt{3}\,j}$$

Question 18. *(4 marks)*
Find the area of the figure illustrated below. It is made up of two equal circular sectors and an equilateral triangle (a triangle with 3 equal sides)



SECTOR AREA:

$$A = \tfrac{1}{2} r^2 \theta$$
$$= \frac{1}{2}(570)^2(\pi/3)$$
$$= 54150\pi \text{ mm}^2$$

2 SECTORS $108\,300\pi \text{ mm}^2$

TRIANGLE : $h = \dfrac{285}{\tan(\pi/6)} = 493.63 \text{mm}$

$$\text{Area} = \frac{b \cdot h}{2} = 140\,684.55 \text{ mm}^2$$

TOTAL : $\boxed{480\,919 \text{ mm}^2}$