NAME: SOLUTIONS

TEST 1

Dawson College
Applied Math (201-943-DW)
Date: Oct 7th 2011
Instructor: E. Richer
This test is marked out of 75 marks

Question 1. *(2 marks each = 12 marks)*
TRUE or FALSE: For each of the following statements, indicate whether it is true or false. If the statement is false, give a correction to the statement.

(a) $-5^{-1} = 5$

$-5^{-1} = -\frac{1}{5}$ FALSE

(b) $(-64)^{\frac{2}{3}} = -16$

$(-64)^{2/3} = (\sqrt[3]{64})^2 = (-4)^2 = 16$ FALSE

(c) $\frac{1}{\frac{1}{x^{-1}}} = x$

$\frac{1}{\frac{1}{x^{-1}}} = \frac{1}{x}$ FALSE

(d) $2 - x(3+x) = -6x - 2x^2$

$2 - 3x - x^2 = 2 - x(3+x)$ FALSE

(e) $-2^{-3} = (-2)^{-3}$ TRUE

(f) $\sqrt[4]{16} = 2$ TRUE

Question 2. *(3 marks each = 15 marks)*
Using the laws of exponents, simplify the following expressions. There should be **no negative exponents in your final answer**.

(a) $(-3^2(-x)^4)^3(3^4x^6)$

$= -3^6 x^{12} 3^4 x^6$

$= \boxed{-3^{10} x^{18}}$

(b) $\frac{8}{-64x^{-5}}$

$= -\frac{8x^5}{64} = \boxed{-\frac{x^5}{8}}$

(c) $\frac{2^2a^4b^{-2}}{(2a)^4b^{-1}}$

$= \frac{2^2a^4b}{2^4a^4b^2} = \boxed{\frac{1}{2^2b}}$

(d) $(4^{-\frac{1}{2}}a^3b^{-1})^{-4}$

$= 4^2a^{-12}b^4$

$= \boxed{\frac{4^2b^4}{a^{12}}}$

(e) $(3^3x^{-\frac{3}{2}}y^{-4})^4(9x^{\frac{1}{2}}y^{-2})$

$= (3^{12}x^{-6}y^{-16})(3^2x^{1/2}y^{-2})$

$= 3^{14}x^{-11/2}y^{-18}$

$= \boxed{\frac{3^{14}}{x^{11/2}y^{18}}}$

Question 3. *(2 marks each = 10 marks)*
Evaluate the following.

(a) $\sqrt[3]{-8}$ = $\boxed{-2}$

(b) $4^{-0.5}$ = $4^{-\frac{1}{2}} = \frac{1}{4^{\frac{1}{2}}} = \frac{1}{\sqrt{4}} = \boxed{\frac{1}{2}}$

(c) $-9^{\frac{3}{2}}$ = $-(9)^{\frac{3}{2}} = -(\sqrt{9})^3 = -(3)^3 = \boxed{-27}$

(d) $\sqrt[4]{16}$ = $\boxed{2}$

(e) $\sqrt[3]{-125}$ = $\boxed{-5}$

Question 4. *(6 marks)*
Ten litres of a fuel-mixture containing 60% oil and 40% gasoline is required for an engine. To make the fuel-mixture, the only products available are a tank of pure gasoline and a tank of 80% oil and 20% gasoline fuel-mixture. How much pure gasoline and how much of the 80% oil and 20% fuel-mixture must be combined in order to get the required product?

REQUIRED : 60% OIL, 40% GAS } 10 LITRES = 6L OIL, 4L GAS

WE HAVE

(A) 80% OIL, 20% GAS — X LITRES

&

(B) 100% GAS — 10 − X LITRES

AMOUNT OF OIL IN (A) + AMOUNT OF OIL IN (B) = AMOUNT OF OIL IN FINAL MIX

$0.8x + 0(10-x) = 6$

$0.8x = 6$

$x = 7.5\text{ L}$

SO 2.5 L OF GASOLINE MUST BE ADDED TO 7.5 L OF 80% OIL - 20% GAS MIX TO MAKE 10L OF DESIRED PRODUCT.

Question 5. *(3 marks each = 12 marks)*
Isolate the specified variable in the given expressions:

(a) $2a+5=2-(3+a)$ (isolate a)

$2a+5 = 2-3-a$

$2a+5 = -1-a$

$3a = -6$ $\quad \boxed{a=-2}$

(b) $\frac{2}{x}+4y=\frac{y}{5}-3$ (isolate x)

MULTIPLY BOTH SIDES by $5x$

$5x\left(\frac{2}{x}+4y\right) = 5x\left(\frac{y}{5}-3\right)$

$10 + 20xy = xy - 15x$

ALL 'x' TERMS ON ONE SIDE

$20xy - xy + 15x = -10$

$19xy + 15x = -10$

$x(19y+15) = -10$

$\boxed{x = -10/19y+15}$

(c) $a^2b-3(a+b)=\frac{3}{a}-2b$ (isolate b)

MULTIPLY EVERYTHING by a

$a^3b - 3a(\ +b) = 3 - 2ab$

$a^3b - 3a^2 - 3ab = 3 - 2ab$

$a^3b - ab = 3 + 3a^2$

$b(a^3-a) = 3+3a^2$

$b = \frac{3+3a^2}{a^3-a} = \boxed{\frac{3(1+a^2)}{a(a^2-1)}}$

(d) $x^{-1}y^2-\frac{3y}{2}=24$ (isolate x)

$\frac{y^2}{x} - \frac{3y}{2} = 24$ MULTIPLY by '2x'

$2x\left(\frac{y^2}{x} - \frac{3y}{2}\right) = 2x(24)$

$2y^2 - 3xy = 48x$

$2y^2 = 48x + 3xy$

$2y^2 = x(48+3y)$

$\boxed{x = \frac{2y^2}{48+3y}}$

Question 6. *(6 marks)*

Solve the system of equations.

$x-3y+3z=-4$

$2x+3y-z=15$

$4x-3y-z+=19$

ISOLATE x IN EQUATION (1)

$x = -4+3y-3z$

SUBSTITUTE IN EQUATIONS (2) & (3)

$2(-4+3y-3z)+3y-z=15$

$-8+6y-6z+3y-z=15$

$9y-7z=23$ (N2)

$4(-4+3y-3z)-3y-z=19$

$-16+12y-12z-3y-z=19$

$9y-13z=35$ (N3)

ISOLATE y IN (N2)

$9y = 23+7z$

$y = \frac{23+7z}{9}$

SUB IN (N3)

$9y-13z=35$

$9\left(\frac{23+7z}{9}\right)-13z=35$

$23+7z-13z=35$

$-6z=12$

$\boxed{z=-2}$ $\longrightarrow$ $y = \frac{23+7z}{9} = \frac{23+7(-2)}{9} = \frac{23-14}{9} = \frac{9}{9} = 1$

$\boxed{y=1}$

$\longrightarrow$ $x = -4+3y-3z$

$= -4+3(1)-3(-2)$

$= -4+3+6$

$= 5$

$\boxed{x=5}$

SOLUTION $\boxed{(x,y,z) = (5,1,-2)}$.

Question 7. *(2 marks each = 14 marks)*
Perform the following operations on polynomials and simplify the answer.

(a) $4a^5 - 2a + 3(a-2) - 5a^4(a+2)$

$= 4a^5 - 2a + 3a - 6 - 5a^5 - 10a^4$

$= \boxed{-a^5 - 10a^4 + a - 6}$

(b) $a - 2(1-a) + 3(-3a-5)$

$= a - 2 + 2a - 9a - 15$

$= \boxed{-6a - 17}$

(c) $(2x-3)(4-x)$

$= 8x - 2x^2 - 12 + 3x$

$= \boxed{-2x^2 + 11x - 12}$

(d) $2y(y-1)(5-3y)$

$= (2y^2 - 2y)(5 - 3y)$

$= 10y^2 - 6y^3 - 10y + 6y^2$

$= \boxed{-6y^3 + 16y^2 - 10y}$

(e) $\frac{2xy-4x^2y}{xy}$

$= \frac{2xy}{xy} - \frac{4x^2y}{xy}$

$= \boxed{2 - 4x}$

(f) $2xy^2 - x(x-3(4-y))$

$= 2xy^2 - x(x-12+3y)$

$= \boxed{2xy^2 - x^2 + 12x - 3xy}$

(g) $(x^3 - 4x + 3) \div (x-1)$

$$
\begin{array}{r|l}
x-1 & x^2+x-3 \\
\hline
 & x^3+0x^2-4x+3 \\
 & -(x^3-x^2) \\
\hline
 & \quad x^2-4x+3 \\
 & \quad -(x^2-x) \\
\hline
 & \quad\quad -3x+3 \\
 & \quad\quad -(-3x+3) \\
\hline
 & \quad\quad\quad 0
\end{array}
$$

ANSWER : $\boxed{x^2+x-3}$

CHECK: $(x-1)(x^2+x-3)$

$= x^3+x^2-3x-x^2-x+3$

$= x^3-4x+3$ CORRECT