NAME: SOLUTIONS

# TEST 2

Dawson College
Applied Math (201-943-DW)
Date: Nov. 10th 2011
Instructor: E. Richer
**This test is marked out of 45 marks**

**Question 1.** *(5 marks)*
In 1982 the population in Boboville was 1300 people, 20 years later the population was 15 000 people. Suppose the population of Boboville can be described by the exponential growth formula $P = P_0 e^{kt}$. What will be Boboville's population in the year 2030.

1982 $t = 0$ $P_0 = 1300$
$t = 20$ $P = 15\,000$
2030 $t = 48$ $P = ?$

FIRST FIND K:

$$P = 1300 e^{kt}$$
$$15\,000 = 1300 e^{20k}$$
$$\frac{15000}{1300} = e^{20k}$$
$$\ln\left(\frac{15\,000}{1300}\right) = \ln e^{20k}$$
$$\ln\left(\frac{15000}{1300}\right) = 20k \longrightarrow k = \frac{\ln\left(\frac{15\,000}{1300}\right)}{20} \approx 0.1223$$

$t = 48$:

$$P = 1300 e^{0.1223t}$$
$$P = 1300 e^{0.1223(48)}$$
$$\approx \boxed{460\,708 \text{ people}}$$

**Question 2.** Factor completely the following expressions: *(6 marks)*

(a) $2x^4 - 2x^3 - 12x^2$

$= 2x^2(x^2 - x - 6)$

$= \boxed{2x^2(x-3)(x+2)}$

(b) $m^3 + 27n^3$ SUM OF CUBES

$= \boxed{(m+3n)(m^2 - 3mn + 9n^2)}$

(c) $12x^2 + 5x - 3$

$mn = -36$

$m+n = 5$

9 & -4

$= 12x^2 - 4x + 9x - 3$

$= 4x(3x-1) + 3(3x-1)$

$= \boxed{(3x-1)(4x+3)}$

**Question 3.** *(5 marks)*

The current $i$ (in $A$) in a certain electric circuit is given by $i = 16(1 - e^{-250t})$, where $t$ is the time (in $s$).

(a) What is the current in the circuit after 5 milliseconds? *(2 marks)*

$i = 16(1 - e^{-250(0.005)}) \approx \boxed{11.4A}$

(b) When will the current in the circuit be 15.999$A$? *(3 marks)*

$i = 16(1 - e^{-250t})$

$15.999 = 16(1 - e^{-250t})$

$\frac{15.999}{16} = 1 - e^{-250t}$

$0.0000625 = e^{-250t}$

$\ln(0.0000625) = \ln e^{-250t}$

$\ln(0.0000625) = -250t$

$\boxed{t = 0.038 \text{ seconds}}$

**Question 4.** Solve the following equations: *(12 marks)*

(a) $7y^2 = 12y - 5$ *(2 marks)*

$7y^2 - 12y + 5 = 0$

$7y^2 - 7y - 5y + 5 = 0$

$7y(y-1) - 5(y-1) = 0$

$(y-1)(7y-5) = 0$ $\boxed{y = 1 \ \& \ y = 5/7}$

(b) $25x^2 - 4 = 0$ *(2 marks)*

$(5x-2)(5x+2) = 0$

$\boxed{x = \pm 2/5}$

(c) $\log_2 x + \log_2(x+2) = 0$ *(3 marks)*

$\log_2(x^2+2x) = 0$

$x^2 + 2x = 2^0$

$x^2 + 2x - 1 = 0 \longrightarrow x = \frac{-2 \pm \sqrt{4-4(-1)}}{2} = \frac{-2 \pm \sqrt{8}}{2} = \frac{-2 \pm 2\sqrt{2}}{2}$

$= \boxed{-1 \pm \sqrt{2}}$

BUT only $-1 + \sqrt{2}$ is a solution

(d) $3e^{x+1} = 5$ *(2 marks)*

$e^{x+1} = 5/3$

$\ln e^{x+1} = \ln 5/3$

$x + 1 = \ln 5/3 \longrightarrow \boxed{x = \ln 5/3 - 1 \approx -0.489}$

(e) $2(14^x) = 4(3^x)$ *(3 marks)*

$\ln(2(14^x)) = \ln(4(3^x))$

$\ln 2 + \ln 14^x = \ln 4 + \ln 3^x$

$\ln 2 + x \ln 14 = \ln 4 + x \ln 3$

$x \ln 14 - x \ln 3 = \ln 4 - \ln 2$

$x(\ln 14 - \ln 3) = \ln 4 - \ln 2$

$\boxed{x = \frac{\ln 4 - \ln 2}{\ln 14 - \ln 3}}$

**Question 5.** *(3 marks)*
Solve for $y$ in terms of $x$:
$\log_5 x + \log_5 y = \log_5 3 + 1$

$\log_5 y = \log_5 3 - \log_5 x + 1$

$\log_5 y = \log_5 (3/x) + 1$

$\log_5 y = \log_5 (3/x) + \log_5 5$

$\log_5 y = \log_5 (15/x)$

$\boxed{y = 15/x}$

**Question 6.** *(4 marks)*
(a) Graph the function $y = f(x) = -x^2 - x + 12$. Your graph must include the $x$-intercept(s), $y$-intercept and the vertex.

Y INTERCEPT $x=0$ $y=12$ $(0,12)$

X INTERCEPTS $0 = -x^2 - x + 12$

$x^2 + x - 12 = 0$

$(x+4)(x-3) = 0$

$x = -4$ & $x = 3$ $(-4,0)$ & $(3,0)$

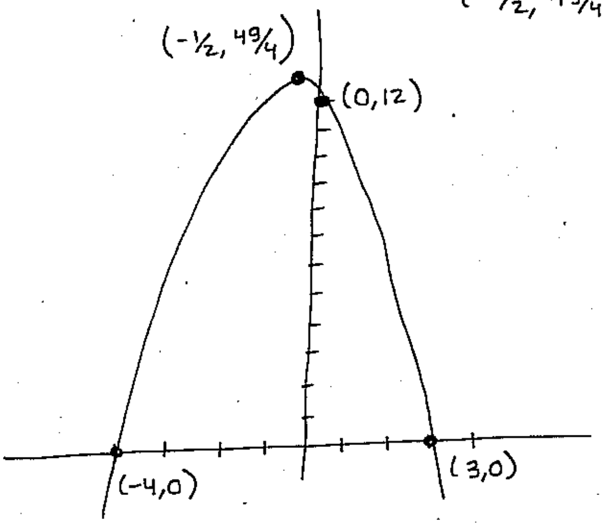
VERTEX:

$x = -b/2a = 1/2(-1) = -1/2$

$y = -(-1/2)^2 - (-1/2) + 12$

$= -1/4 + 1/2 + 12 = -1/4 + 2/4 + 48/4 = 49/4$

$(-1/2, 49/4)$




(b) State the domain and range of $f(x)$.

DOMAIN $\mathbb{R}$

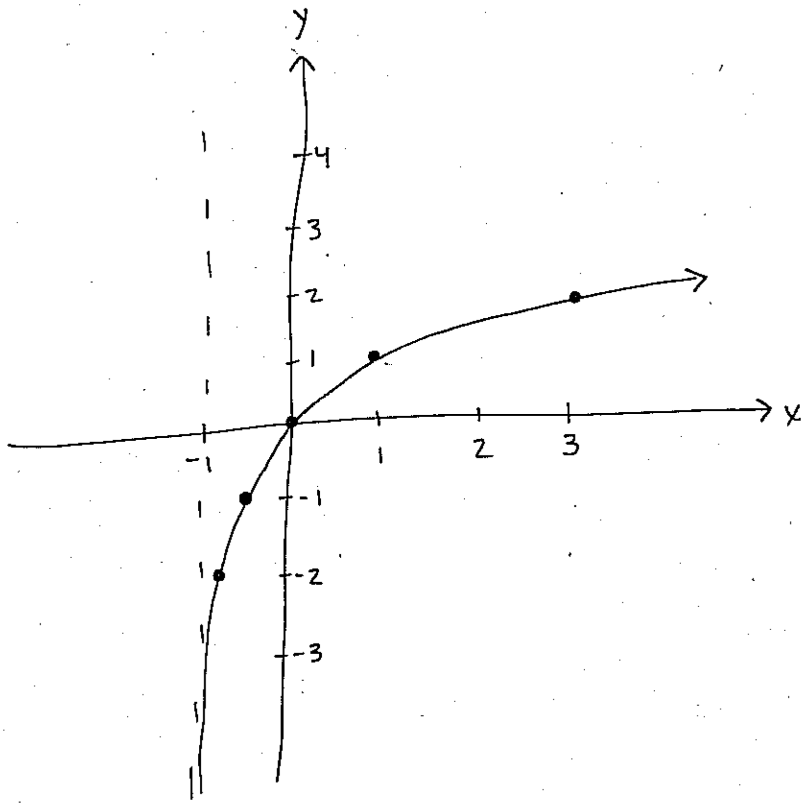
RANGE $(-\infty, 49/4]$

**Question 7.** *(6 marks)*
Consider the equation $y = \log_2(x+1)$.
(a) Give the exponential form of the equation. *(1 mark)*

$$2^y = x+1$$

(b) Graph the function $y = f(x) = \log_2(x+1)$. *(3 marks)*

| $y$ | $x = 2^y - 1$ |
|---|---|
| $-2$ | $-3/4$ |
| $-1$ | $-1/2$ |
| $0$ | $0$ |
| $1$ | $1$ |
| $2$ | $3$ |




(c) Give the domain and range of the function $f$. *(2 marks)*

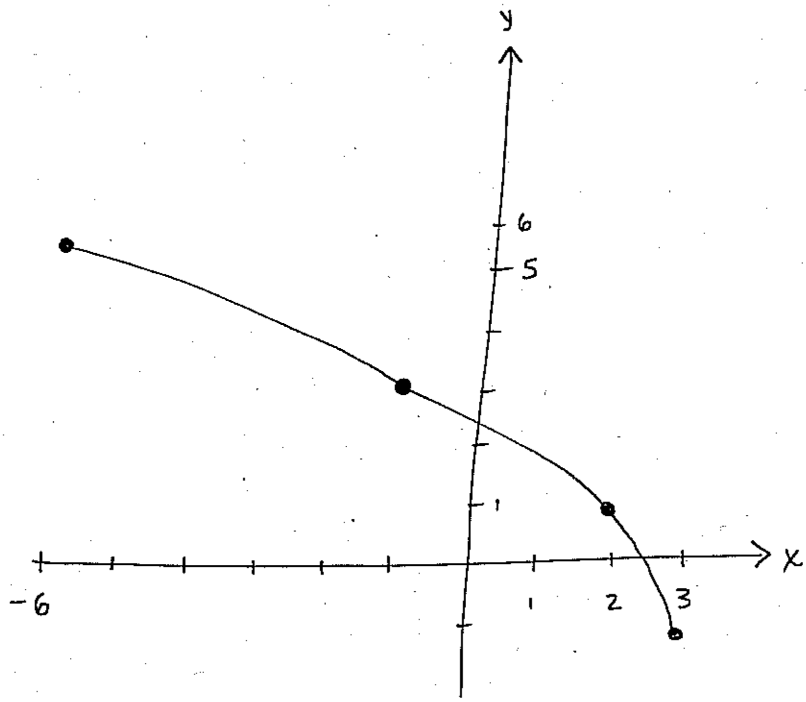
DOMAIN: $(-1, \infty)$

RANGE: $\mathbb{R}$

**Question 8.** *(4 marks)*
(a) Graph the function $y = f(x) = 2\sqrt{3-x} - 1$. *(2 marks)*

VERTEX $x = 3$

| $x$ | $y$ |
|---|---|
| 3 | -1 |
| 2 | 1 |
| -1 | 3 |
| -6 | 5 |




(b) Give the domain and range of the function $f$. *(2 marks)*

DOMAIN $(-\infty, 3]$

RANGE $[-1, \infty)$