

NAME: Solutions

TEST 3

Dawson College

Applied Math (201-943-DW)

Date: Dec. 9th 2011

Instructor: E. Richer

This test is marked out of 50 marks

Question 1. (3 marks)

Express the measure of each angle in terms of degrees

$$(a) \frac{2\pi}{5} \quad \frac{2\pi}{5} \cdot \frac{180^\circ}{\pi} = \boxed{72^\circ}$$

$$(b) \frac{4\pi}{3} \quad \frac{4\pi}{3} \cdot \frac{180^\circ}{\pi} = \boxed{240^\circ}$$

$$(c) \frac{5\pi}{4} \quad \frac{5\pi}{4} \cdot \frac{180^\circ}{\pi} = \boxed{225^\circ}$$

Question 2. (3 marks)

Express the given angle measurements in radian measure **in terms of π** .

$$(a) 60^\circ \quad 60^\circ \cdot \frac{\pi}{180^\circ} = \boxed{\frac{\pi}{3}}$$

$$(b) 330^\circ \quad 330^\circ \cdot \frac{\pi}{180^\circ} = \boxed{\frac{11\pi}{6}}$$

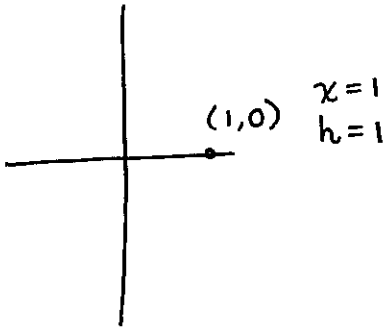
$$(c) 150^\circ \quad 150^\circ \cdot \frac{\pi}{180^\circ} = \boxed{\frac{5\pi}{6}}$$

Question 3. (10 marks)

For each equation below, find all values of θ within $0^\circ \leq \theta \leq 360^\circ$.

(a) $\cos \theta - 1 = 0$

$$\cos \theta = 1$$

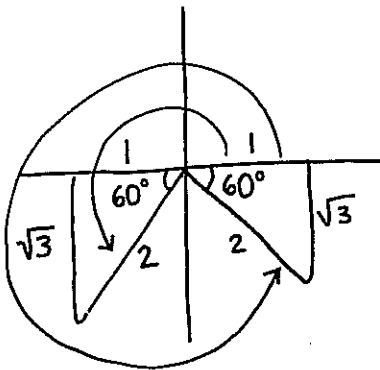


$$\theta_1 = 0^\circ$$

$$\theta_2 = 360^\circ$$

(b) $2 \sin \theta + \sqrt{3} = 0$

$$\sin \theta = -\frac{\sqrt{3}}{2}$$

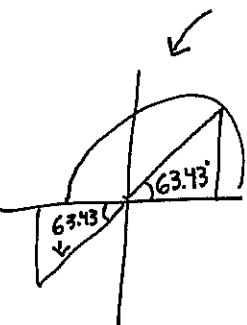


$$\theta = 240^\circ \text{ \& } 300^\circ$$

$$(c) \tan^2 \theta - \tan \theta - 2 = 0$$

$$(\tan \theta - 2)(\tan \theta + 1) = 0$$

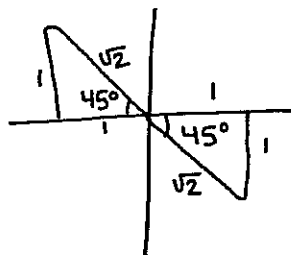
$$\tan \theta = 2 \quad \tan \theta = -1$$



$$\theta_1 = 63.43^\circ$$

$$\theta_2 = 180^\circ + 63.43^\circ$$

$$= 243.43^\circ$$



$$\theta_3 = 135^\circ$$

$$\theta_4 = 315^\circ$$

$$(d) 2 \sec \theta - 7 = 0 \text{ with the restriction that } \sin \theta < 0$$

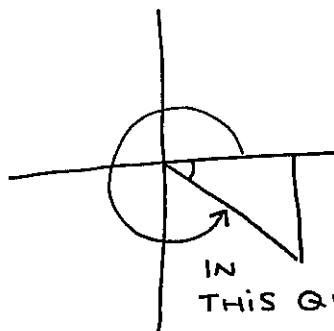
$$\sec \theta = 7/2$$

$$\cos \theta = 2/7$$

$$\cos^{-1}(2/7) = 73.39^\circ$$

$$\theta = 360^\circ - 73.39^\circ$$

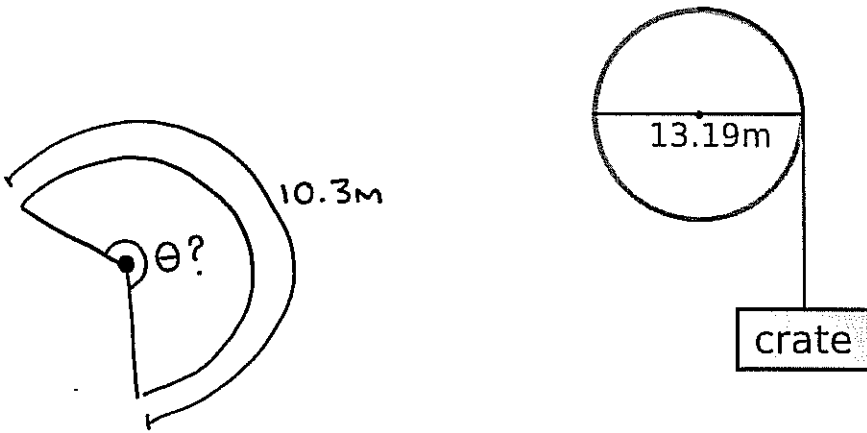
$$= 286.61^\circ$$



IN
THIS QUADRANT
 $\cos \theta > 0$
 $\sin \theta < 0$

Question 4. (3 marks)

Through what angle does the drum in the figure below turn in order to lower the crate 10.3m from its current position?



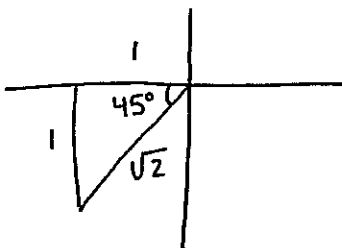
$$r_{\text{radius}} = 13.19/2 = 6.595 \text{ m}$$

$$\theta = s/r = \frac{10.3}{6.595} = 1.56 \text{ rad} = 89.5^\circ$$

Question 5. (3 marks)

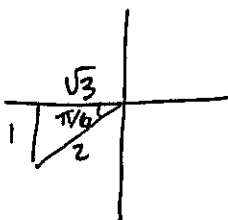
Using "special triangles" find the **EXACT VALUE** of the following:

(a) $\sin(225^\circ)$



$$\begin{aligned} \sin 225^\circ &= y/h \\ &= \boxed{-1/\sqrt{2}} \end{aligned}$$

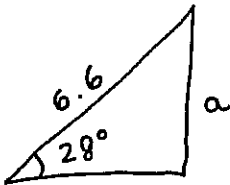
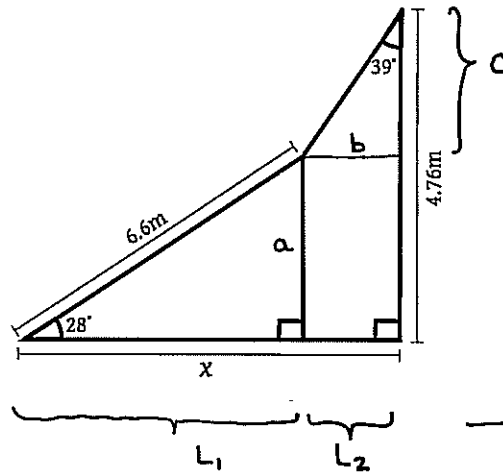
(b) $\tan(7\pi/6)$



$$\tan 7\pi/6 = \frac{y}{x} = \frac{-1}{-2} = \boxed{1/2}$$

Question 6. (5 marks)

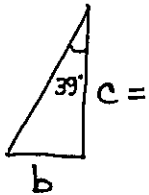
Find the length x in the diagram given below.



$$\textcircled{1} \sin 28^\circ = \frac{a}{6.6}$$

$$a = 6.6 \sin 28^\circ = 3.09 \text{ m}$$

$$\textcircled{2} c = 4.76 \text{ m} - a = 1.67 \text{ m}$$



$$\textcircled{3} \tan 39^\circ = \frac{b}{c}$$

$$\begin{aligned} b &= 1.67 \tan 39^\circ \\ &= 1.35 \text{ m} \\ &= L_2 \end{aligned}$$

$$\textcircled{4} \cos 28^\circ = \frac{L_1}{6.6}$$

$$L_1 = 5.82 \text{ m}$$

$$\begin{aligned} \textcircled{5} \quad x &= L_1 + L_2 \\ &= 5.82 + 1.35 \\ &= \boxed{7.17 \text{ m}} \end{aligned}$$

Question 7. (2 marks each = 10 marks)

Perform the indicated operations, expressing all answers in simplest rectangular form.

(a) $(12 + 3j) + (-3 - 6j)$

$$\boxed{9 - 3j}$$

(b) $(-4 - 2j) - \sqrt{-36}$

$$= -4 - 2j - 6j$$

$$= \boxed{-4 - 8j}$$

(c) $j(3 - 2j) - (j^3)(5 + j)$

$$= 3j - 2j^2 - (-j)(5 + j)$$

$$= 3j + 2 + 5j + j^2$$

$$= \boxed{1 + 8j}$$

$$(d) \frac{2+3j}{-1-4j} \cdot \frac{(-1+4j)}{(-1+4j)} = \frac{-2+8j-3j+12j^2}{1-16j^2} = \frac{-14+5j}{17} = \boxed{-\frac{14}{17} + \frac{5}{17}j}$$

$$(e) \frac{j^4 - 2j^2 + 4j^3}{2-j} = \frac{1+2-4j}{2-j} \cdot \frac{(2+j)}{(2+j)} = \frac{(3-4j)(2+j)}{(2-j)(2+j)} = \frac{6+3j-8j-4j^2}{4-j^2}$$

$$= \frac{10-5j}{5}$$

$$= \boxed{2-j}$$

Question 8. (3 marks each = 6 marks)

Give the polar and exponential form of each of the complex numbers.

(a) $-1 + \sqrt{3}j$

POLAR FORM : $2\cos 120^\circ + 2\sin 120^\circ j$

EXPONENTIAL FORM : $2 e^{2\pi/3 j}$

(b) $-2 - 2j$

POLAR FORM : $2\sqrt{2}(\cos 225^\circ + \sin 225^\circ j)$

EXPONENTIAL FORM : $2\sqrt{2} e^{5\pi/4 j}$

Question 9 (3 marks)

Find the values of x and y for which the equation is valid:

$$(3 - 2j)(1 + j) = x + yj^9$$

$$j^9 = j$$

$$3 + 3j - 2j - 2j^2 = x + yj$$

$$5 + j = x + yj$$

$$x = 5$$

$$y = 1$$

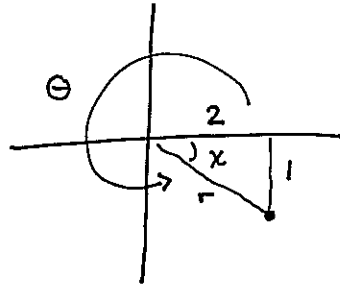
Question 10. (4 marks)

Change the number to exponential form and then perform the indicated operations.

Express the final result in rectangular and polar forms.

$$(2-j)^5$$

$$2-j :$$



$$r^2 = 1^2 + 2^2$$

$$r = \sqrt{5}$$

$$x = \tan^{-1}(1/2) \approx 26.6^\circ$$

$$\theta = 360^\circ - x = 333.4^\circ = 5.82 \text{ rad}$$

$$2-j = \sqrt{5} e^{5.82j}$$

$$(2-j)^5 = (\sqrt{5} e^{5.82j})^5$$

$$= 55.9 e^{29.1j}$$

$$(2-j)^5 = 55.9 \cos 29.1 + 55.9 \sin 29.1 j$$
$$= \boxed{-37.9 - 41.1j}$$