Dawson College: **Calculus III**: 201-BZF-05 S01 December 2, 2011

Last Name: SOLUTIONS

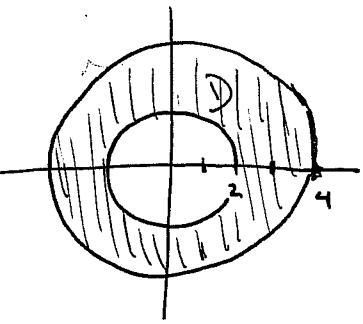
First Name: ____________

Student ID: ____________

Quiz 10

Question 1. *(5 marks)* Use polar coordinates to find the volume of the solid inside the sphere $x^2+y^2+z^2=16$ and outside the cylinder $x^2+y^2=4$.

$x^2+y^2+z^2=16$



$D = \{(r,\theta) \mid 0 \le \theta \le 2\pi, \quad 2 \le r \le 4\}$

so $V = 2\iint_D \sqrt{16-x^2-y^2}\, dA$

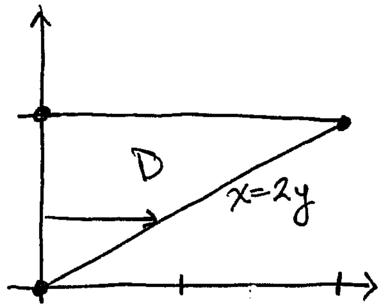
$= 2\int_0^{2\pi}\int_2^4 \sqrt{16-r^2}\; r\, dr d\theta$

$= 2\int_0^{2\pi} d\theta \int_2^4 r(16-r^2)^{1/2} dr = 2\left[\theta\right]_0^{2\pi}\left[-\frac{1}{3}(16-r^2)^{3/2}\right]_2^4$

$= -\frac{2}{3}(2\pi)(0-12^{3/2}) = \frac{4\pi}{3}(12\sqrt{12}) = 32\sqrt{3}\pi$

Question 2. *(5 marks)* Find the surface area of the part of the surface $z = 1 + 3x + 2y^2$ that lies above the triangle with verticies $(0,0)$, $(0,1)$ and $(2,1)$.

LET $f(x,y) = 1 + 3x + 2y^2$ $\quad \therefore D = \{(x,y) \mid 0 \le x \le 2y,\ 0 \le y \le 1\}$



$$\therefore A(S) = \iint_D \sqrt{1 + (3)^2 + (4y)^2}\, dA$$

$$= \int_0^1 \int_0^{2y} \sqrt{10 + 16y^2}\, dx\, dy$$

$$= \int_0^1 \sqrt{10 + 16y^2}\, [x]_{x=0}^{x=2y}\, dy = \int_0^1 2y\sqrt{10 + 16y^2}\, dy$$

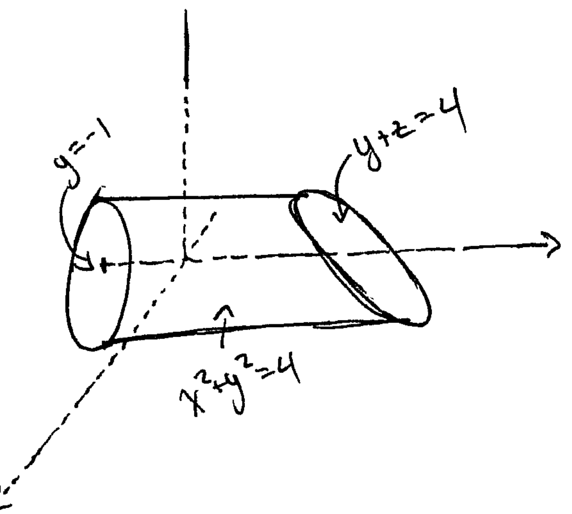
$$= 2 \cdot \frac{1}{32} \cdot \frac{2}{3} (10 + 16y^2)^{3/2} \Big]_0^1 = \frac{1}{24}\left(26^{3/2} - 10^{3/2}\right)$$

Question 3. *(10 marks)* Find

$$\iiint_E f(x,y,z)\,dV$$

where $f(x,y,z) = 1$ and E is the region enclosed by the cylinder $x^2+z^2 = 4$ and the planes $y = -1$ and $y+z = 4$. Note that this integral gives you the volume of E.

(Hints: Sketch a graph of E. To see what the plane $y+z = 4$ looks like, look at the projection on the yz-plane ($x = 0$) and think of what the projestion would look like on any plane $x = a$. For this integral you will probably have to use the tigonometric substitution $x = 2\sin\theta$.)



$\therefore E = \{(x,y,z) \mid -2 \le x \le 2, -\sqrt{4-x^2} \le z \le \sqrt{4-x^2}, -1 \le y \le 4-z\}$

$$\therefore V = \int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}}\int_{-1}^{4-z} dy\,dz\,dx$$

$$= \int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} (4-z+1)\,dz\,dx$$

$$= \int_{-2}^{2}\left[5z - \frac{1}{2}z^2\right]_{z=-\sqrt{4-x^2}}^{z=\sqrt{4-x^2}} = \int_{-2}^{2} 10\sqrt{4-x^2}\,dx$$

$$\boxed{\begin{aligned}\text{LET } x &= 2\sin\theta \\ dx &= 2\cos\theta\,d\theta\end{aligned}}$$

$$= \int_{x=-2}^{x=2} 10\sqrt{4-4\sin^2\theta}\,2\cos\theta\,d\theta$$

$$= \int_{x=-2}^{x=2} 40\cos^2\theta\,d\theta = 40\int_{x=-2}^{x=2} \frac{1}{2} + \frac{1}{2}\cos 2\theta\,d\theta = 40\left[\frac{1}{2}\theta + \frac{1}{4}\sin 2\theta\right]_{x=-2}^{x=2}$$

$$= 40\left[\frac{1}{2}\theta + \frac{1}{4}\cdot 2\sin\theta\cos\theta\right]_{x=-2}^{x=2}$$

$$= \left[20\sin^{-1}\left(\frac{x}{2}\right) + 20\cdot\frac{x}{2}\cdot\frac{\sqrt{4-x^2}}{2}\right]_{x=-2}^{x=2}$$

$$= 20\sin^{-1}(1) + 5(2)\sqrt{4-4} - 20\sin^{-1}(-1)\cdot 5(2)\sqrt{0}$$

$$= 20\cdot\frac{\pi}{2} - 20\left(-\frac{\pi}{2}\right) = \underline{20\pi}$$

$\sin\theta = \frac{x}{2}$