Dawson College: Calculus III: 201-BZF-05 S01 September 9, 2011

Last Name: SOLUTIONS

First Name:

Student ID:

Quiz 2

Question 1. *(5 marks)* Determine if the series is convergent or divergent. If it is convergent find its sum.

$$\sum_{n=1}^{\infty} \frac{1+2^n}{3^n} = \sum_{n=1}^{\infty}\left[\left(\frac{1}{3}\right)^n + \left(\frac{2}{3}\right)^n\right]$$

NOW $\displaystyle\sum_{n=1}^{\infty}\left(\frac{1}{3}\right)^n = \sum_{n=1}^{\infty}\frac{1}{3}\left(\frac{1}{3}\right)^{n-1} = \frac{1/3}{1-1/3} = \frac{1/3}{2/3} = 1/2$ (GEOMETRIC, $r = 1/3$)

$$\sum_{n=1}^{\infty}\left(\frac{2}{3}\right)^n = \sum_{n=1}^{\infty}\frac{2}{3}\left(\frac{2}{3}\right)^{n-1} = \frac{2/3}{1-2/3} = \frac{2/3}{1/3} = 2 \text{ (GEOMETRIC, } r = 2/3\text{)}$$

$$\therefore \sum_{n=1}^{\infty}\left[\left(1/3\right)^n + \left(2/3\right)^n\right] = \sum_{n=1}^{\infty}\left(\frac{1}{3}\right)^n + \sum_{n=1}^{\infty}\left(\frac{2}{3}\right)^n = 1/2 + 2 = 5/2$$

$\therefore$ THE SERIES CONVERGES.

Question 2. *(5 marks)* Determine if the series is convergent or divergent.

$$\sum_{n=2}^{\infty} \frac{1}{n\ln n}$$

LET $f(x) = \dfrac{1}{x\ln x} \Rightarrow f'(x) = \dfrac{-[\ln x + x \cdot 1/x]}{(x\ln x)^2} = \dfrac{-\ln x - 1}{(x\ln x)^2} < 0$

FOR $x \geq 2$

$\therefore$ f IS A POSITIVE, DECREASING AND CONTINUOUS AT $[2, \infty)$

$$\int_2^{\infty} f(x)\,dx = \int_2^{\infty}\frac{1}{x\ln x}\,dx = \lim_{t\to\infty}\int_2^{t}\frac{1}{x\ln x}\,dx$$

$$= \lim_{t\to\infty}\left[\ln(\ln x)\right]_2^t$$

$$= \lim_{t\to\infty}\left[\ln(\ln t) - \ln(\ln 2)\right]$$

$$= \infty$$

LET $u = \ln x$ $du = 1/x\,dx$

$$\int \frac{1}{x\ln x}\,dx = \int \frac{1}{u}\,du$$

$$= \ln u + c = \ln(\ln x) + C$$

$\therefore$ THE SERIES DIVERGES BY INTEGRAL TEST.