

Last Name: SOLUTIONS

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## Quiz 3

Question 1. (5 marks) Test the following series for convergence or divergence.

(a) (5 marks)  $\sum_{n=1}^{\infty} \frac{3^n n^2}{n!}$       LET  $a_n = \frac{3^n n^2}{n!}$

$$\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n \rightarrow \infty} \left| \frac{3^{n+1} (n+1)^2}{(n+1)!} \cdot \frac{n!}{3^n n^2} \right| = \lim_{n \rightarrow \infty} \frac{\cancel{3^n} \cdot 3 (n+1)^2 \cancel{n!}}{(n+1) \cdot \cancel{n!} \cdot \cancel{3^n} \cdot n^2}$$

$$= \lim_{n \rightarrow \infty} 3 \cdot \left( \frac{n+1}{n} \right)^2 \cdot \frac{1}{n+1} = 3 \cdot 1 \cdot 0 = 0 < 1$$

$\therefore$  THE SERIES CONVERGES (ABSOLUTELY) BY RATIO TEST

(b) (5 marks)  $\sum_{n=1}^{\infty} (\sqrt[n]{2} - 1)^n$       LET  $a_n = (\sqrt[n]{2} - 1)^n$

$$\lim_{n \rightarrow \infty} \sqrt[n]{|a_n|} = \lim_{n \rightarrow \infty} \sqrt[n]{|(\sqrt[n]{2} - 1)^n|} = \lim_{n \rightarrow \infty} (\sqrt[n]{2} - 1)$$

$$= \lim_{n \rightarrow \infty} (2^{1/n} - 1) = 2^0 - 1 = 0 < 1$$

$\therefore$  THE SERIES CONVERGES (ABSOLUTELY)  
BY THE ROOT TEST.