Dawson College: **Calculus III**: 201-BZF-05 S01 September 30, 2011

Last Name: SOLUTIONS.

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Quiz 4

Question 1. *(9+1 marks)* Use a power series to approximate the following definite integral to six decimal places. Clearly state any theorems that you are using.

(a) $\displaystyle\int_0^{0.4} \ln(1+x^4)\,dx$

$$\ln(1+x) = \int \frac{1}{1-(-x)}\,dx = \int \sum_{n=0}^{\infty} (-x)^n\,dx = \underline{\int \sum_{n=0}^{\infty} (-1)^n x^n\,dx} = \sum_{n=0}^{\infty} (-1)^n \frac{x^{n+1}}{n+1} + c \quad \boxed{\text{on } 1<x<-1}$$

BUT $C=0$ SINCE $\ln(1+0) = 0 + c$

$0 = c$

$$\Rightarrow \ln(1+x^4) = \sum_{n=0}^{\infty} \frac{(-1)^n (x^4)^{n+1}}{n+1} = \sum_{n=0}^{\infty} \frac{(-1)^n x^{4n+4}}{n+1} \quad \text{on } 1<x<-1$$

$$\therefore \int \ln(1+x^4)\,dx = \int \sum_{n=0}^{\infty} \frac{(-1)^n x^{4n+4}}{n+1}\,dx = \sum_{n=0}^{\infty} \frac{(-1)^n x^{4n+5}}{(n+1)(4n+5)} + C$$

$$\Rightarrow \int_0^{0.4} \ln(1+x^4)\,dx = \sum_{n=0}^{\infty} \frac{(-1)^n (0.4)^{4n+5}}{(n+1)(4n+5)} - \sum_{n=0}^{\infty} 0$$

$$\frac{(0.4)^5}{1\cdot 5} - \frac{(0.4)^9}{2\cdot 9} + \frac{(0.4)^{13}}{3\cdot 13} - \frac{(0.4)^{17}}{4\cdot 17} + \frac{(0.4)^{21}}{5\cdot 21} - \frac{(0.4)^{25}}{6\cdot 25} + \cdots$$

NOW $b_2 = \dfrac{(0.4)^{13}}{3\cdot 13} \approx 0.00000017207 < 0.0000002$

$\therefore$ BY ALTERNATING SERIES ESTIMATION THM* $|s - s_1| < b_2 < 0.0000002$

$\Rightarrow \quad s \approx s_1 = \dfrac{(0.4)^5}{1\cdot 5} - \dfrac{(0.4)^9}{2\cdot 9} = 0.002033$ ACCURATE TO SIX DECIMAL PLACES.

* SINCE $\displaystyle\lim_{n\to\infty} b_n = \lim_{n\to\infty} \frac{(0.4)^{4n+5}}{(n+1)(4n+5)} = 0$

AND $b_{n+1} = \dfrac{(0.4)^{4n+9}}{(n+2)(4n+9)} < \dfrac{(0.4)^{4n+5}}{(n+1)(4n+5)} = b_n$

(b) Can you use this method to approximate $\displaystyle\int_0^{3.1} \ln(1+x^4)\,dx$ to six decimal places?

Why or why not? If it is possible you do not have to approximate.

NO, SINCE THE SERIES WE USED AS AN ANTIDERIVATIVE OF $\ln(1+x^4)$ HAS RADIUS 1 WITH CENTRE 0 SO IT MAY NOT BE AN ANTIDERIVATIVE AT $x = 3.1$.