Dawson College: Calculus III: 201-BZF-05 S01 October 21, 2011

Last Name: SOLUTIONS

First Name: ______

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Quiz 6

Question 1. *(5 marks)* Given $\mathbf{r}(t) = \langle t^2, \frac{2}{3}t^3, t\rangle$, find the vectors **T**, **N**, and **B** at the point on the curve $(1, \frac{2}{3}, 1)$.

$\vec{r}'(t) = \langle 2t, 2t^2, 1\rangle \Rightarrow |\vec{r}'(t)| = \sqrt{4t^2+4t^4+1} = \sqrt{(2t^2+1)^2}$
$= 2t^2+1$

$\therefore \vec{T}(t) = \dfrac{\vec{r}'(t)}{|\vec{r}'(t)|} = \dfrac{\langle 2t, 2t^2, 1\rangle}{2t^2+1} \Rightarrow \vec{T}(1) = \langle \frac{2}{3}, \frac{2}{3}, \frac{1}{3}\rangle$

$\vec{T}'(t) = -4t(2t^2+1)^{-2}\langle 2t, 2t^2, 1\rangle + (2t^2+1)^{-1}\langle 2, 4t, 0\rangle$
$= 2(2t^2+1)^{-2}\langle 1-2t^2, 2t, -2t\rangle$

$\vec{N}(t) = \dfrac{\vec{T}'(t)}{|T'(t)|} = \dfrac{\langle 1-2t^2, 2t, -2t\rangle}{1+2t^2} \Rightarrow N(1) = \langle -\frac{1}{3}, \frac{2}{3}, -\frac{2}{3}\rangle$

$\therefore \vec{B}(1) = \vec{T}(1)\times\vec{N}(1) = \langle -\frac{2}{3}, \frac{1}{3}, \frac{2}{3}\rangle$

Question 2. *(5 marks)* Find the velocity and postition vectors of a particle that has the acceleration vector $\mathbf{a}(t) = 2\mathbf{i} + 6t\ \mathbf{j} + 12t^2\mathbf{k}$ and the given initial velocity $\mathbf{v}(0) = \mathbf{i}$ and initial position $\mathbf{r}(0) = \mathbf{j} - \mathbf{k}$.

$\vec{v}(t) = \int \vec{a}(t)dt = 2t\vec{i} + 3t^2\vec{j} + 4t^3\vec{k} + \vec{C}$

$\therefore \vec{i} = \vec{v}(0) = \vec{C} \Rightarrow \vec{v}(t) = (2t+1)\vec{i} + 3t^2\vec{j} + 4t^3\vec{k}$

$\vec{r}(t) = \int \vec{v}(t)dt = (t^2+t)\vec{i} + t^3\vec{j} + t^4\vec{k} + \vec{D}$

But $\vec{j} - \vec{k} = \vec{r}(0) = \vec{D}$

$\therefore \vec{r}(t) = (t^2+t)\vec{i} + (t^3+1)\vec{j} + (t^4-1)\vec{k}$