· $f_x(x,y) = \frac{1}{y}\cos\left(\frac{x}{y}\right)$ , $f_y(x,y) = 1 + \left(-\frac{x}{y^2}\right)\cos\left(\frac{x}{y}\right)$

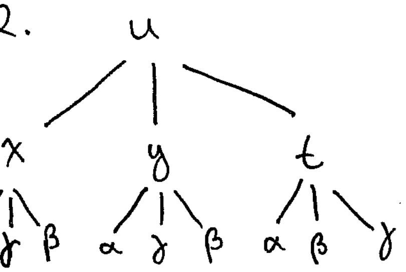
$$L(x,y) = f(0,3) + f_x(0,3)\cdot(x-0) + f_y(0,3)\cdot(y-3)$$

$$= \left[3+\sin(0)\right] + \left[\tfrac{1}{3}\cos(0)\right]x + \left[1 + 0\cdot\cos(0)\right](y-3)$$

$$= 3 + \tfrac{1}{3}x + y - 3$$

$$= \tfrac{1}{3}x + y$$

2.



·. $\frac{\partial u}{\partial \alpha} = \frac{\partial u}{\partial x}\cdot\frac{\partial x}{\partial \alpha} + \frac{\partial u}{\partial y}\cdot\frac{\partial y}{\partial \alpha} + \frac{\partial u}{\partial t}\cdot\frac{\partial t}{\partial \gamma}$

$$= e^{ty}\cdot 2\alpha\beta + xte^{ty}\cdot 0 + xye^{ty}\cdot \gamma^2$$

WHEN $\alpha=-1$, $\beta=2$, $\gamma=1$
$x=2$, $y=4$, $t=-1$

WHEN $\alpha=-1$, $\beta=2$, $\gamma=1$

$$\frac{\partial u}{\partial \alpha} = e^{-4}\cdot 2(-1)(2) + 0 + (2(4)e^{-4})((1)^2)$$

$$= -4e^{-4} + 8e^{-4} = 4e^{-4}$$

$$\frac{\partial u}{\partial \beta} = \frac{\partial u}{\partial x}\cdot\frac{\partial x}{\partial \beta} + \frac{\partial u}{\partial y}\cdot\frac{\partial y}{\partial \beta} + \frac{\partial u}{\partial t}\cdot\frac{\partial t}{\partial \beta}$$

$$= e^{ty}\alpha^2 + xte^{ty}\cdot(2\beta\gamma) + xye^{ty}(0)$$

WHEN $\alpha=-1$, $\beta=2$, $\gamma=1$

$$\frac{\partial u}{\partial \beta} = e^{-4}(-1)^2 + (2)(-1)e^{-4}\cdot 2(2)(1) + 0 = -7e^{-4}$$

$$\frac{\partial u}{\partial \gamma} = \frac{\partial u}{\partial x}\cdot\frac{\partial x}{\partial \gamma} + \frac{\partial u}{\partial y}\cdot\frac{\partial y}{\partial \gamma} + \frac{\partial u}{\partial t}\cdot\frac{\partial t}{\partial \gamma}$$

$$= e^{ty}(0) + xte^{ty}(\beta^2) + xye^{ty}(2\gamma\alpha)$$

WHEN $\alpha=-1,\ \beta=2,\ \gamma=1$

$$\frac{\partial u}{\partial \gamma} = 0 + 2(-1)e^{-4}\cdot(2)^2 + (2)(4)e^{-4}\cdot(2(1)(-1))$$

$$= -8e^{-4} - 16e^{-4}$$

$$= -24e^{-4}$$

3. $f_r = \dfrac{s}{1+(rs)^2}$ , $g_s = \dfrac{r}{1+(rs)^2}$

$$\therefore\ \nabla f = \left\langle \frac{s}{1+(rs)^2}, \frac{r}{1+(rs)^2} \right\rangle$$

$$\vec{u} = \frac{\vec{u}}{\|\vec{u}\|} = \frac{1}{5\sqrt{5}}\langle 5, 10\rangle = \left\langle \frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}} \right\rangle$$

$$\therefore\ D_{\vec{u}} f(2,1) = \left\langle \frac{1}{1+2^2}, \frac{2}{1+2^2} \right\rangle \cdot \left\langle \frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}} \right\rangle$$

$$= \frac{1}{5\sqrt{5}} + \frac{4}{5\sqrt{5}} = \frac{1}{\sqrt{5}}$$