QUIZ 9 SOLUTIONS

1. $\nabla f = \left\langle \frac{qr}{1+(pqr)^2}, \frac{pr}{1+(pqr)^2}, \frac{pq}{1+(pqr)^2} \right\rangle$

$\nabla f(1,2,1) = \left\langle \frac{2}{5}, \frac{1}{5}, \frac{2}{5} \right\rangle$

$\therefore$ THE MAXIMUM RATE OF CHANGE IS $|\nabla f(1,2,1)| = \sqrt{\frac{4}{25}+\frac{1}{25}+\frac{4}{25}}$
$= \frac{3}{5}$

IN THE DIRECTION OF $\left\langle \frac{2}{5}, \frac{1}{5}, \frac{2}{5} \right\rangle$

2. $f_x = -2xe^y$, $f_y = (2y+y^2-x^2)e^y$

$f_{xx} = -2e^y$, $f_{xy} = -2xe^y$, $f_{yy} = (2+4y+y^2-x^2)e^y$.

$f_x = 0 \Rightarrow x = 0 \Rightarrow$
$f_y = 0 \Rightarrow (2y+y^2)e^y = 0 \Rightarrow y(2+y) = 0 \Rightarrow y = 0, -2$

$\therefore (0,0)$ & $(-2,0)$ ARE THE C.P.

$D(0,0) = (-2)(2) - (0)^2 = -4 < 0$ SO THERE IS A SADDLE POINT AT $(0,0)$

$D(0,-2) = (-2e^{-2})(-2e^{-2}) - 0^2 = 4e^{-4} > 0$

AND $f_{xx}(0,-2) = -2e^{-2} < 0$ SO $f(0,-2) = 4e^{-2}$ IS A LOCAL MAXIMUM.

3. LET x, y, z BE THE DIMENSIONS OF THE BOX

THE SURFACE AREA $= 2xy + 2xz + 2yz$

IT $xyz = 1000 \Rightarrow z = \dfrac{1000}{xy}$ SO WE MINIMIZE

$$f(x,y) = 2xy + 2x\left(\frac{1000}{xy}\right) + 2y\left(\frac{1000}{xy}\right)$$

$$\Rightarrow f_x = 2y - \frac{2000}{x^2} \quad \text{AND} \quad f_y = 2x - \frac{2000}{y^2}$$

$f_x = 0 \Rightarrow y = \dfrac{1000}{x^2}$ SO $f_y = 0 \Rightarrow x - \dfrac{x^4}{1000} = 0$

$\therefore x^3 = 1000$ OR 0 BUT $x \neq 0$ SINCE $xyz = 1000$

$\therefore x = 10$

THE SURFACE AREA HAS A MINIMUM BUT NO MAXIMUM SO IT MUST OCCUR AT A CRITICLE POINT.

$\therefore$ THE BOX WITH MINIMUM SURFACE AREA HAS DIMENSIONS

$x = 10$ cm, $y = \dfrac{1000}{10^2} = 10$ cm AND $z = \dfrac{1000}{10^2} = 10$cm