

Last Name: SOLUTIONS

First Name: _____

Student ID: _____

Quiz 4 (A)

Question 1. (4 marks) Evaluate the following limit:

$$\lim_{x \rightarrow 1} \frac{\sqrt{x}-1}{x-1} = \lim_{x \rightarrow 1} \frac{\sqrt{x}-1}{\sqrt{x}-1} \cdot \frac{\sqrt{x}+1}{\sqrt{x}+1} = \lim_{x \rightarrow 1} \frac{x - \sqrt{x} + \sqrt{x} - 1}{(\sqrt{x}-1)(\sqrt{x}+1)}$$

$$= \lim_{x \rightarrow 1} \frac{x-1}{(\sqrt{x}-1)(\sqrt{x}+1)} = \lim_{x \rightarrow 1} \frac{1}{\sqrt{x}+1} = \frac{1}{\sqrt{1}+1} = \frac{1}{2}$$

Question 2. (6 marks) Determine where the following function is continuous. Explain your answer.

$$f(x) = \begin{cases} -5x+3 & \text{if } x < 0 \\ 4 & \text{if } x = 0 \\ 4x^2 - x + 3 & \text{if } x > 0 \end{cases}$$

$f(x) = -5x+3$ when $x < 0$ (A POLYNOMIAL) so f is continuous when $x < 0$
 $f(x) = 4x^2 - x + 3$ when $x > 0$ (A POLYNOMIAL) so f is continuous when $x > 0$

AT $x=0$

1) $f(0) = 4$

2) $\lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^-} (-5x+3) = -5(0)+3 = 3$

$$\lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0^+} (4x^2 - x + 3) = 4(0)^2 - (0) + 3 = 3$$

$$\therefore \lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0^-} f(x) \Rightarrow \lim_{x \rightarrow 0} f(x) \text{ EXISTS } (=3)$$

3) $\lim_{x \rightarrow 0} f(x) = 3 \neq 4 = f(0)$ (CONDITION 3 FAILS)

$\therefore f(x)$ IS DISCONTINUOUS AT $x=0$

$\therefore f$ IS CONTINUOUS EVERYWHERE EXCEPT $x=0$.