Dawson College: **Calculus II**: 201-NYA-05 S17

October 7, 2011

Last Name: SOLUTIONS

First Name: ______

Student ID: ______

Test 1

The length of the test is 1hr and 45min. You may use a nonprogrammable, nongraphing scientific calculator. Remember to clearly show all of your work and to **use correct notation**.

Question 1. Evaluate the following:

(a) *(1 mark)* $\sin\left(\frac{15\pi}{6}\right) = \sin\left(\frac{5\pi}{2}\right) = 1$

(b) *(1 mark)* $\sec\left(-\frac{5\pi}{4}\right) = \frac{1}{\cos\left(-\frac{5\pi}{4}\right)} = \frac{1}{-\sqrt{2}/2} = -\frac{2}{\sqrt{2}}$ or $-\sqrt{2}$

(d) *(2 mark)* $\arcsin\left(\sin\left(\frac{4\pi}{3}\right)\right) = \arcsin\left(-\frac{\sqrt{3}}{2}\right) = -\pi/3$

(f) *(2 mark)* Find θ on $0 \leq \theta \leq 2\pi$ given $\csc\theta = \frac{2}{\sqrt{3}}$ $\Rightarrow \frac{1}{\sin\theta} = \frac{2}{\sqrt{3}}$

$\Rightarrow \sin\theta = \frac{\sqrt{3}}{2}$

$\therefore \theta = \frac{\pi}{3}, \frac{2\pi}{3}$

Question 2. Evaluate the following limits:

(a) (*2 marks*) $\lim_{x \to 3^+} \frac{x^2-9}{5x+2} = \frac{(3)^2-9}{5(3)+2} = \frac{0}{17} = 0$

(b) (*4 marks*) $\lim_{x \to 2} \frac{3-\sqrt{2x+5}}{x-2} = \lim_{x \to 2} \frac{3-\sqrt{2x+5}}{x-2} \cdot \frac{3+\sqrt{2x+5}}{3+\sqrt{2x+5}}$

$= \lim_{x \to 2} \frac{9 - 3\cancel{\sqrt{2x+5}} + 3\cancel{\sqrt{2x+5}} - (2x+5)}{(x-2)(3+\sqrt{2x+5})} = \lim_{x \to 2} \frac{9-2x-5}{(x-2)(3+\sqrt{2x+5})}$

$= \lim_{x \to 2} \frac{4-2x}{(x-2)(3+\sqrt{2x+5})} = \lim_{x \to 2} \frac{-2\cancel{(x-2)}}{\cancel{(x-2)}(3+\sqrt{2x+5})}$

$= \lim_{x \to 2} \frac{-2}{3+\sqrt{2x+5}} = \frac{-2}{3+\sqrt{2 \cdot 2+5}} = \frac{-2}{3+3} = -\frac{2}{6} = -\frac{1}{3}$

(c) (*4 marks*) $\lim_{x \to 1} \frac{x^3-1}{x^2+x-2} = \lim_{x \to 1} \frac{\cancel{(x-1)}(x^2+x+1)}{\cancel{(x-1)}(x+2)}$

$= \lim_{x \to 1} \frac{x^2+x+1}{x+2} = \frac{1^2+1+1}{1+2} = \frac{3}{3} = 1$

(d) (*4 marks*) $\lim\limits_{x \to -\infty} \dfrac{5x^3 + 9x^2 - x + 8}{-2x^3 + 3x^2 + 1} = \lim\limits_{x \to -\infty} \dfrac{5x^3/x^3 + 9x^2/x^3 - x/x^3 + 8/x^3}{-2x^3/x^3 + 2x^2/x^3 + 1/x^3}$

$$= \lim_{x \to -\infty} \frac{5 + 9/x - 1/x^2 + 8/x^3}{-2 + 2/x + 1/x^3} = \frac{5 + 0 - 0 + 0}{-2 + 0 + 0}$$

$$= -\frac{5}{2}$$

(d) (*4 marks*) $\lim\limits_{x \to \infty} \dfrac{2x^4 + x^3 - x}{4x^2 - 7x + 2} = \lim\limits_{x \to \infty} \dfrac{\frac{2x^4}{x^2} + \frac{x^3}{x^2} - \frac{x}{x^2}}{\frac{4x^2}{x^2} - \frac{7x}{x^2} + \frac{2}{x^2}}$

$$= \lim_{x \to \infty} \frac{2x^2 + x - 1/x}{4 + 7/x + 2/x^2} = \infty \qquad (\text{D.N.E.})$$

Question 3. *(5 marks)* For which values of x is the following function continuous? (Do not graph this function). Clearly explain your reasoning.

$$f(x) = \begin{cases} 3x^2 + 2x & \text{if } x < 2 \\ 5x + 6 & \text{if } x \geq 2 \end{cases}$$

WHEN $x < 2$, f IS A POLYNOMIAL $\therefore$ CONTINUOUS

WHEN $x > 2$, f IS A POLYNOMIAL $\therefore$ CONTINUOUS

AT $x = 2$

1) $f(2) = 5(2) + 6 = 16$

2) $\lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} (3x^2 + 2x) = 3(2)^2 + 2(2) = 16$

$\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} (5x + 6) = 5(2) + 6 = 16$

$\therefore \lim_{x \to 2^-} f(x) = \lim_{x \to 2^+} f(x) \Rightarrow \lim_{x \to 2} f(x)$ EXISTS (AND $= 16$)

3) $\lim_{x \to 2} f(x) = 16 = f(2)$

$\therefore$ f IS CONTINUOUS AT $x = 2$

$\therefore$ f IS CONTINUOUS EVERYWHERE.

Question 4. *(4 marks)* Let $f(x) = x^2 + 3x - 2$. Find $f'(x)$ using the limit definition of the derivative.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{[(x+h)^2 + 3(x+h) - 2] - [x^2 + 3x - 2]}{h}$$

$$= \lim_{h \to 0} \frac{x^2 + 2xh + h^2 + 3x + 3h - 2 - x^2 - 3x - 2}{h}$$

$$= \lim_{h \to 0} \frac{2xh + h^2 + 3h}{h}$$

$$= \lim_{h \to 0} \frac{h(2x + h + 3)}{h}$$

$$= \lim_{h \to 0} (2x + h + 3)$$

$$= 2x + 0 + 3$$

$$= 2x + 3$$

Question 5. Find the derivatives of the following functions. Do not simplify.

(a) *(4 marks)* $f(x) = 2\sqrt{x} - \frac{3}{x^3} + 7\cos x + 3$ $= 2x^{1/2} - 3x^{-3} + 7\cos x + 3$

$$f'(x) = 2\left(\tfrac{1}{2}x^{-1/2}\right) - 3\left(-3x^{-4}\right) + 7(-\sin x) + 0$$

$$= \frac{1}{x^{1/2}} + \frac{9}{x^4} - 7\sin x$$

(b) *(4 marks)* $g(t) = \frac{t^3 + 2t}{t^2 + 6t - 9}$

$$g'(t) = \frac{\frac{d}{dt}\left[t^3+2t\right]\cdot(t^2+6t-9) - (t^3+2t)\cdot\frac{d}{dt}\left[t^2+6t-9\right]}{(t^2+6t-9)^2}$$

$$= \frac{(3t^2+2)(t^2+6t-9) - (t^3+2t)(2t+6)}{(t^2+6t-9)^2}$$

(c) *(4 marks)* $h(x) = (x^2+3x-3)(x^5+2x^3+5)\left(x^2 - \frac{1}{x}\right)$

$$h'(x) = \frac{d}{dx}\left[x^2 - \frac{1}{x}\right]\cdot(x^2+3x-3)(x^5+2x^3+5) + \frac{d}{dx}\left[(x^2+3x-3)(x^5+2x^3+5)\right]\cdot\left(x^2-\frac{1}{x}\right)$$

$$= \left(2x + \frac{1}{x^2}\right)(x^2+3x-3)(x^5+2x^3+5) + \left[\frac{d}{dx}\left[x^2+3x-3\right]\cdot(x^5+2x^3+5) + \frac{d}{dx}\left[x^5+2x^3+5\right]\cdot(x^2+3x-3)\right]\cdot\left(x^2-\frac{1}{x}\right)$$

$$= \left(2x + \frac{1}{x^2}\right)(x^2+3x-3)(x^5+2x^3+5) + \left[(2x+3)(x^5+2x^3+5) + (5x^4+6x^2)(x^2+3x-3)\right]\left(x^2-\frac{1}{x}\right)$$

Question 6. *(5 marks)* Let $f(x) = x^3 + 9x^2 + 24x - 2$.
(a) Find the point(s) on the graph of f where the tangent line to the curve is horizontal.
(b) Find the equation of the tangent line to graph of f at $x = 1$.

a) HORIZONTAL TANGENT LINE $\Leftrightarrow$ SLOPE OF TANGENT LINE $= 0$
$\Leftrightarrow f'(x) = 0$

$$f'(x) = 3x^2 + 18x + 24 = 0$$
$$3(x^2 + 6x + 8) = 0$$
$$3(x+4)(x+2) = 0$$
$$\therefore \; x = -2, -4$$

IF $x = -2$, $y = f(-2) = (-2)^3 + 9(-2)^2 + 24(-2) - 2 = -22$

IF $x = -4$, $y = f(-4) = (-4)^3 + 9(-4)^2 + 24(-4) - 2 = -18$

$\therefore$ THE POINTS ON THE GRAPH OF f WHERE THE TANGENT LINE IS HORIZONTAL ARE $(-2, -22)$ AND $(-4, -18)$

b) SLOPE OF TANGENT LINE TO f AT $x = 1$ $= f'(1) = 3(1)^2 + 18(1) + 24 = 45 = m$

$y = f(1) = (1)^3 + 9(1)^2 + 24(1) - 2 = 32$

$$\therefore \; y = mx + b$$
$$32 = 45(1) + b$$
$$-13 = b$$

THE TANGENT LINE TO f AT $x = 1$ IS
$$y = 45x - 13$$

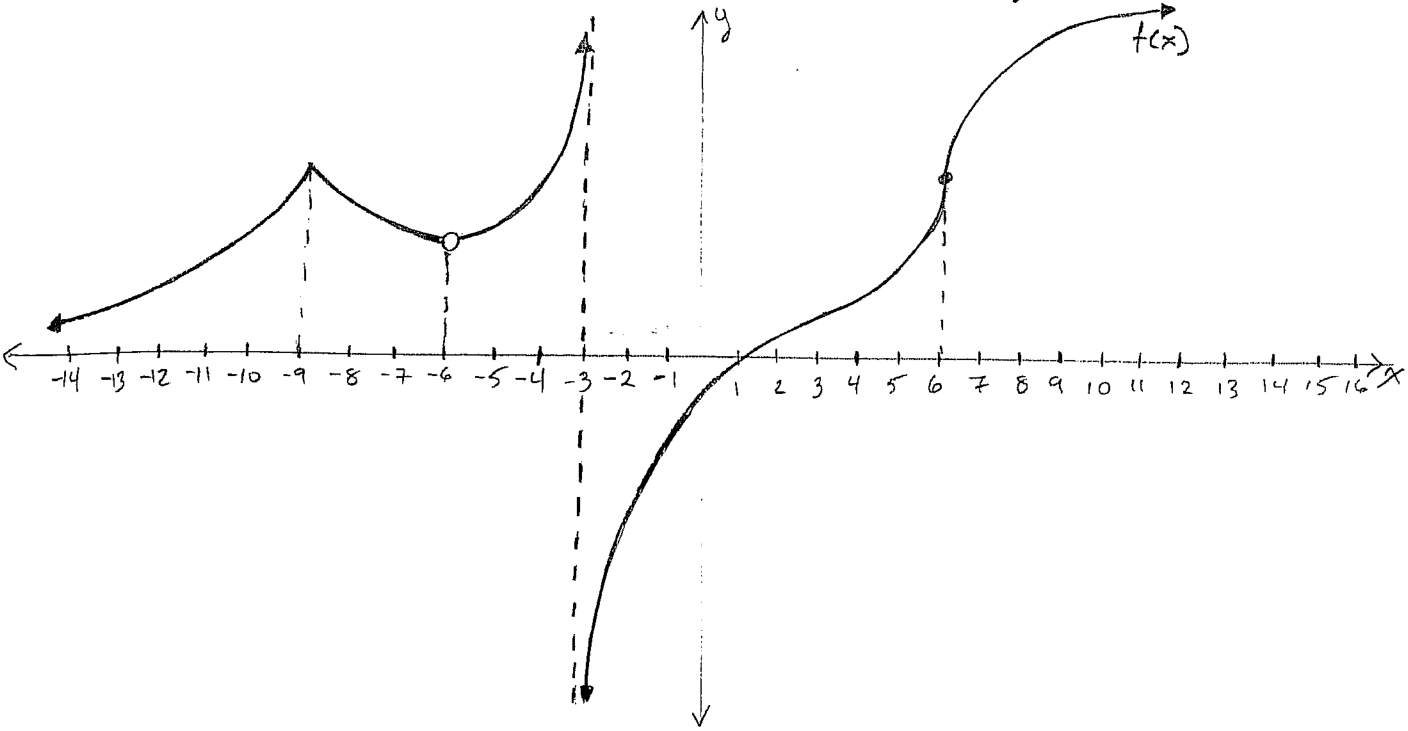
Question 7. *(6 marks)* Given the graph of $f(x)$ find:

(a) All values $x = a$ where $\lim_{x \to a} f(x)$ does not exist.

(b) All values $x = a$ where the function is discontinuous.

For each point state which condition for continuity it fails.

(c) All values $x = a$ where the function is not differentiable and state why.



a) $\lim_{x \to a} f(x)$ D.N.E. WHEN $a = -3$

b) f IS NOT CONTINUOUS AT:

- $x = -6$ SINCE f FAILS CONDITION 1 ($f(-6)$ IS NOT DEFINED)
- $x = -3$ SINCE f FAILS CONDITION 1 (AND 2).
  ($f(-3)$ IS NOT DEFINED AND $\lim_{x \to -3} f(x)$ D.N.E.)

c) f IS NOT DIFFERENTIABLE: AT • $x = -9$ SINCE IT HAS A "CORNER" (CHANGES DIRECTION ABRUPTLY)

- $x = -6$ AND $x = -3$ SINCE f IS NOT CONTINUOUS THERE (SEE PART b)
- $x = 6$ SINCE THE TANGENT LINE AT THAT POINT IS VERTICAL.