Dawson College: **Calculus II**: 201-NYA-05 S17

November 4, 2011

Last Name: SOLUTIONS

First Name: ________________

Student ID: ________________

Test 2

The length of the test is 1hr and 45min. You may use a nonprogrammable, nongraphing scientific calculator. Remember to clearly show all of your work and to **use correct notation**.

Question 1. Find the derivative of the following functions (do not simplify unless otherwise stated):

(a) *(2 marks)* $f(x) = (4x^3 - 5x + 2)^{13} + 3\csc x$

$$f'(x) = 13(4x^3-5x+2)^{12}\cdot(12x^2-5) - 3\csc x\cot x$$

(b) *(4 marks)* $f(x) = (2x^3 - 2)\sqrt{5x^3 + 1}$

$$f'(x) = 6x^2\cdot\sqrt{5x^3+1} + (2x^3-2)\cdot\frac{1}{2}(5x^3+1)^{-1/2}\cdot 15x^2$$

(c) *(5 marks)* $f(x) = \sqrt{\arctan(x^2 - 4x)}$

$$f'(x) = \frac{1}{2}\left(\arctan(x^2-4x)\right)^{-1/2} \cdot \frac{1}{1+(x^2-4x)^2} \cdot (2x-4)$$

(b) *(5 marks)* (Simplify your answer) $f(x) = \dfrac{(3x-1)^4}{(2x+5)^3}$

$$f'(x) = \frac{4(3x-1)^3 \cdot 3\,(2x+5)^3 - (3x-1)^4 \cdot 3(2x+5)^2 \cdot (2)}{[(2x+5)^3]^2}$$

$$= \frac{12(3x-1)^3(2x+5)^3 - 6(3x-1)^4(2x+5)^2}{(2x+5)^6}$$

$$= \frac{6(3x-1)^3(2x+5)^2\left[2(2x+5) - (3x-1)\right]}{(2x+5)^6}$$

$$= \frac{6(3x-1)^3\left[4x+10 - 3x+1\right]}{(2x+5)^4}$$

$$= \frac{6(3x-1)^3\left[x+11\right]}{(2x+5)^4}$$

Question 3. (4 marks) Suppose that $f(x) = g(3x^2+1)$. Find an expression for $f'(x)$. Find $f'(1)$ if $g'(4) = -1$.

$$f'(x) = g'(3x^2+1) \cdot \frac{d}{dx}[3x^2+1]$$
$$= g'(3x^2+1) \cdot 6x$$

$$\therefore f'(1) = g'(3\cdot(1)^2+1) \cdot 6(1)$$
$$= g'(4) \cdot 6$$
$$= (-1)(6)$$
$$= -6$$

Question 4. *(6 marks)* Find the second derivative of the following function. Simplify your final answer.

$$f(x) = (4x^2 - x)^{7/2}$$

$$f'(x) = \frac{7}{2}(4x^2 - x)^{5/2} \cdot (8x-1)$$

$$f''(x) = \frac{7}{2} \cdot \frac{5}{2}(4x^2 - x)^{3/2}(8x-1)^2 + \frac{7}{2}(4x - x)^{5/2} \cdot 8$$

$$= \frac{35}{4}(4x^2 - x)^{3/2} \cdot (8x-1)^2 + 28(4x - x)^{5/2}$$

Question 5. *(6 marks)* James is producing DVDs of his stand-up comedy routine of math jokes from his classes. The weekly total cost of producing these DVDs is given by

$$C(x) = 0.0000002x^3 - 0.05x^2 + 60x + 85$$

Believe it or not, the demand equation for these DVDs is given by

$$p = -0.008x + 25$$

where x is the quatitiy demanded and p is the price per unit in dollars.
(a) Find the revenue function $R(x)$ and the profit function $P(x)$.
(b) Find the marginal profit function $P'(x)$.
(c) Assuming James is selling 500 DVDs, use the marginal profit function to determine if he should try to sell 501 DVDs. Why or why not?

a) $R(x) = xp = x(-0.008x+25) = -0.008x^2 + 25x$

$P(x) = R(x) - C(x) = [-0.008x^2 + 25x] - [0.0000002x^3 - 0.05x^2 + 60x + 85]$

$= -0.0000002x^3 + 0.042x^2 - 35x - 85$

b) $P'(x) = -0.0000006x^2 + 0.084x - 35$

c) $P'(500) = -0.0000006(500)^2 + 0.084(500) - 35$

$= 6.85$

YES HE SHOULD SELL 501 DVDs SINCE HE WOULD MAKE A PROFIT OF (APPROXIMATELY) \$6.85 ON THE 501ST BOOK.

Question 6. *(6 marks)* Using the demand equation from Question 5:

$$p = -0.008x + 25$$

(a) Calculate the elasticity of demand function $E(p)$.
(b) Calculate the elasticity of demand when the price do the DVD is $15. Use the elasticity to determine what should be done to the price to increase revenue?
(c) At what price is the demand for the DVD unitary?

a) $0.008x = -p + 25$

$x = -125p + 3125 = f(p) \Rightarrow f'(p) = -125$

$\therefore E(p) = \dfrac{125p}{3125 - 125p} = \dfrac{p}{25 - p}$

b) $E(15) = \dfrac{15}{10} = \dfrac{3}{2} > 1 \quad \therefore$ DEMAND IS ELASTIC

$\therefore$ PRICE SHOULD BE INCREASED SLIGHTLY TO INCREASE REVENUE

c) $E(p) = \dfrac{p}{25-p} = 1 \Rightarrow p = 25 - p \Rightarrow 2p = 25$

$p = \$12.50$

$\therefore$ DEMAND IS UNITARY WHEN THE PRICE IS $12.50

Bonus (3 marks) In class we discussed the relationship between elasticity and revenue. We determined that if demand is unitary, changing the price slightly will have little effect on the revenue. For most revenue functions, raising or lowering the price too much will result in a decrease in revenue. Draw the graph of a revenue function where this isn't the case. In particular, draw the graph of a revenue function where even though demand is unitary at a certain price, if price is increased enough revenue will be more than at the unitary price. Indicate on you graph the unitary price and the new price where revenue is greater.