Dawson College: **Calculus I**: 201-NYA-05 S17

December 2, 2011

Last Name: SOLUTIONS

First Name: ____________

Student ID: ____________

Test 3

The length of the test is 1hr and 45min. You may use a nonprogrammable, nongraphing scientific calculator. Remember to clearly show all of your work and to **use correct notation**.

Question 1.

(a) *(4 marks)* Find $\frac{dy}{dx}$ given $(x+y)^3 - 4xy = 19$

(b) *(2 marks)* Find the equation of the tangent line to the graph of y at the point $(2,1)$

a) $3(x+y)^2\left(1+\frac{dy}{dx}\right) - 4y - 4x\frac{dy}{dx} = 0$

$3(x+y)^2 + 3(x+y)^2\frac{dy}{dx} - 4y - 4x\frac{dy}{dx} = 0$

$3(x+y)^2\frac{dy}{dx} - 4x\frac{dy}{dx} = 4y - 3(x+y)^2$

$\frac{dy}{dx}\left[3(x+y)^2 - 4x\right] = 4y - 3(x+y)^2$

$\frac{dy}{dx} = \frac{4y - 3(x+y)^2}{3(x+y)^2 - 4x}$

b) $m = \left.\frac{dy}{dx}\right|_{(2,1)} = \frac{4(1) - 3(2+1)^2}{3(2+1)^2 - 4(2)} = \frac{4-27}{27-8} = \frac{-23}{19}$

$y = mx + b$

$1 = 2\left(\frac{-23}{19}\right) + b$

$1 + \frac{46}{19} = b$

$\frac{65}{19} = b$

$y = -\frac{23}{19}x + \frac{65}{19}$

Question 2. *(5 marks)* Suppose the wholesale price of a certain brand of medium-sized eggs p (in dollars/carton) is related to the weekly supply x (in thousand of cartons) by the equation

$$625p^2 - x^2 = 100$$

If the 25 000 cartons of eggs are available at the beginning of a certain week and the supply is falling at a rate of 1000 cartons per week, at what rate is wholesale price changing? (ROUND TO THE NEAREST CENT)

$x = 25, \quad \frac{dx}{dt} = -1, \quad \Rightarrow \quad 625p^2 - (25)^2 = 100$

$625p^2 = 725 \Rightarrow p^2 = 1.16$

$\Rightarrow p = \sqrt{1.16}$

$$\frac{d}{dt}[625p^2] - \frac{d}{dt}[x^2] = \frac{d}{dt}[100]$$

$$1250p\frac{dp}{dt} - 2x\frac{dx}{dt} = 0$$

$$1250\sqrt{1.16}\frac{dp}{dt} - 2(25)(-1) = 0$$

$$1250\sqrt{1.16}\frac{dp}{dt} = -50$$

$$\frac{dp}{dt} = \frac{-50}{1250\sqrt{1.16}} = -0.03714$$

$\therefore$ THE PRICE IS DECREASING BY \$0.04 PER UNIT PER WEEK AT THE TIME IN QUESTION.

Question 3. *(5 marks)* Use the second derivative test to find the relative extrema of $f(x) = \frac{1}{2}x^4 - \frac{2}{3}x^3 - 6x^2 + 1$.

$$f'(x) = 2x^3 - 2x^2 - 12x = 0$$

$$2x(x^2 - x - 6) = 0$$

$$2x(x-3)(x+2) = 0$$

$$\therefore x = -2, 0, 3$$

$$f''(x) = 6x^2 - 4x - 12$$

$f''(-2) = 6(-2)^2 - 4(-2) - 12 = 20 > 0 \Rightarrow f(-2) = -\frac{29}{3}$ IS A RELATIVE MINIMUM.

$f''(0) = -12 < 0 \Rightarrow f(0) = 1$ IS A RELATIVE MAXIMUM.

$f''(3) = 6(3)^2 - 4(3) - 12 = 30 > 0 \Rightarrow f(3) = -\frac{61}{2}$ IS A RELATIVE MINIMUM.

Question 4. *(4 marks)* Find all vertical and horizontal asymptotes of the function

$$f(x) = \frac{-4x^2 - 5}{x^2+8x+16}$$

$x^2+8x+16 = (x+4)(x+4)=0 \Rightarrow x=-4$

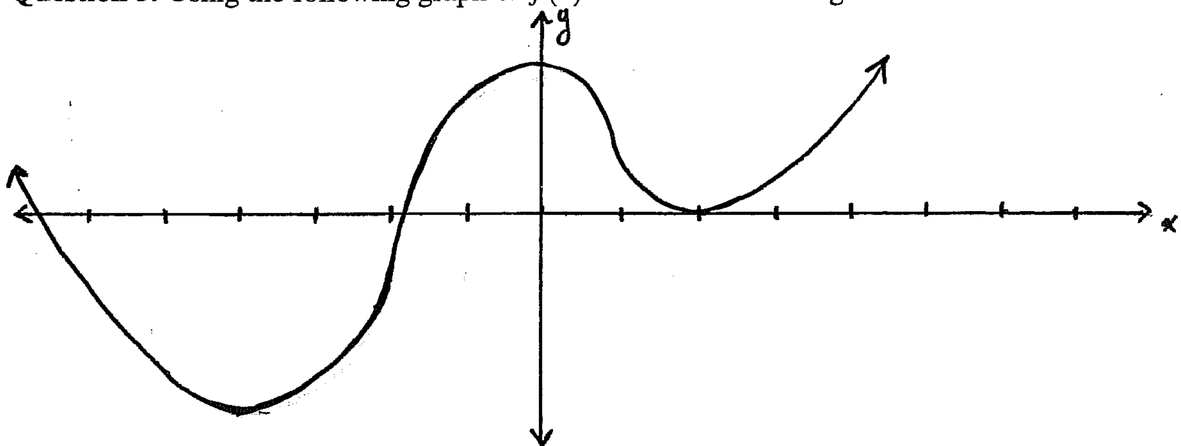
$P(-4) = -4(-4)^2-5 = -69 \neq 0$

$\therefore$ $x=-4$ IS A VERTICAL ASYMPTOTE

$$\lim_{x\to\pm\infty} \frac{-4x^2-5}{x^2+8x+16} = \lim_{x\to\pm\infty} \frac{-4-5/x^2}{1+\frac{8}{x}+\frac{16}{x^2}} = \frac{-4-0}{1+0+0} = -4$$

$\therefore$ $y=-4$ IS A HORIZONTAL ASYMPTOTE

Question 5. Using the following graph of $f(x)$ to find the following information:



(a) *(2 marks)* Intervals where $f'(x) > 0$ and intervals where $f'(x) < 0$ (indicate which is which).

$f'(x) > 0$ ON $(-4, 0)$ AND $(2, \infty)$

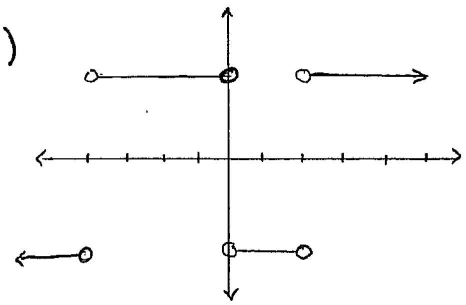
$f'(x) < 0$ ON $(-\infty, -4)$ AND $(0, 2)$

(b) *(2 marks)* x-values where $f'(x) = 0$

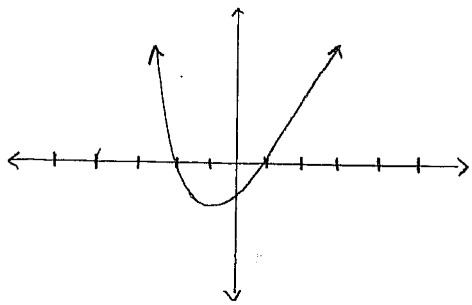
$x = -4, 0, 2$

(c) *(2 marks)* Indicate which of the graphs (**i, ii, iii, iv**) is the graph of $f'(x)$. Justify your answer.

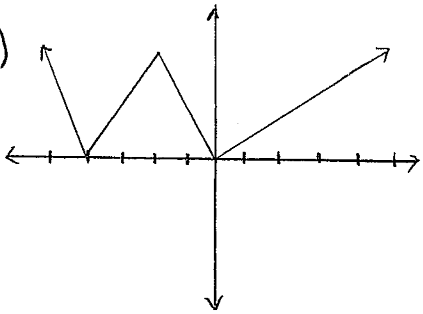
i)



ii)



iii)



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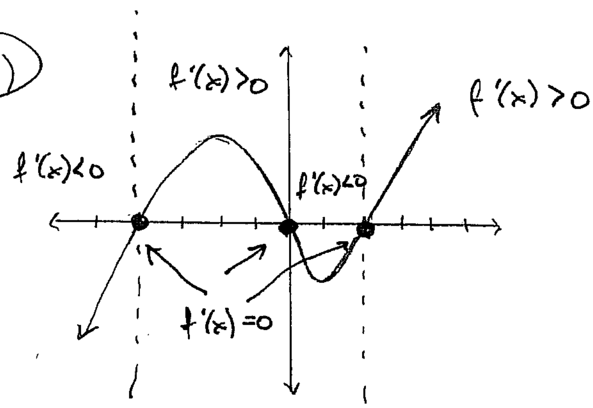

Question 6. Given $f(x) = 4x^3 - x^4$ find:
(a) *(1 marks)* The domain of $f(x)$.

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(b) *(1 marks)* The x and y-intercepts.

x-int: $y=0$

$0 = 4x^3 - x^4$

$0 = x^3(4-x)$

$\therefore x = 0, 4$

$\Rightarrow (0,0), (4,0)$

y-int

$y = 4(0)^3 - 4(0)^4 = 0$

$\therefore (0,0)$

(c) *(3 marks)* Intervals where f is increasing and intervals where f is decreasing.

$f'(x) = 12x^2 - 4x^3$

$f'(x) = 0$

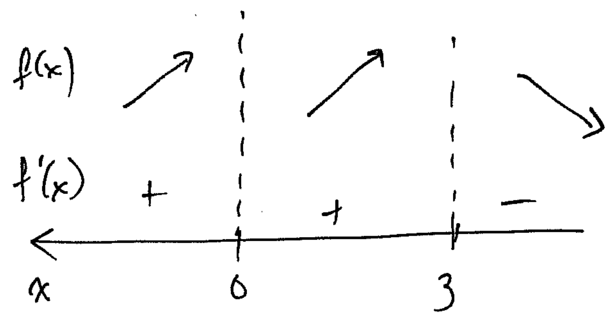
$12x^2 - 4x^3 = 0$

$4x^2(3-x) = 0$

$\therefore x = 0, 3$

$f'(x)$ D.N.E

POLYNOMIAL.



TEST NUMBERS

$x = -1 \quad f'(-1) = 16 > 0$

$x = 1 \quad f'(1) = 8 > 0$

$x = 4 \quad f'(4) = -64 < 0$

$\therefore$ f IS INCREASING ON $(-\infty, 0)$ AND $(0,3)$

f IS DECREASING ON $(3, \infty)$

(d) *(1 marks)* Any relative extrema.

$f(3) = 27$ IS A (RELATIVE) MAXIMUM.

(e) *(3 marks)* Intervals where f is concave upward and intervals where f is concave downward.

$f''(x) = 24x - 12x^2$

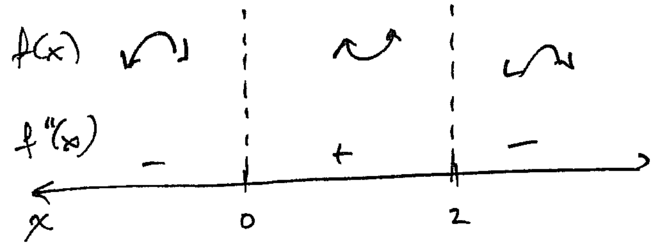
$f''(x) = 0$

$24x - 12x^2 = 0$

$12x(2 - x) = 0$

$x = 0, 2$

$f''(x)$ D.N.E ~~$f''(x)$ D.N.E~~ POLYNOMIAL



TEST NUMBERS

$x = -1: f''(-1) = -36 < 0$

$x = 1: f''(1) = 12 > 0$

$x = 3: f''(3) = -36 < 0$

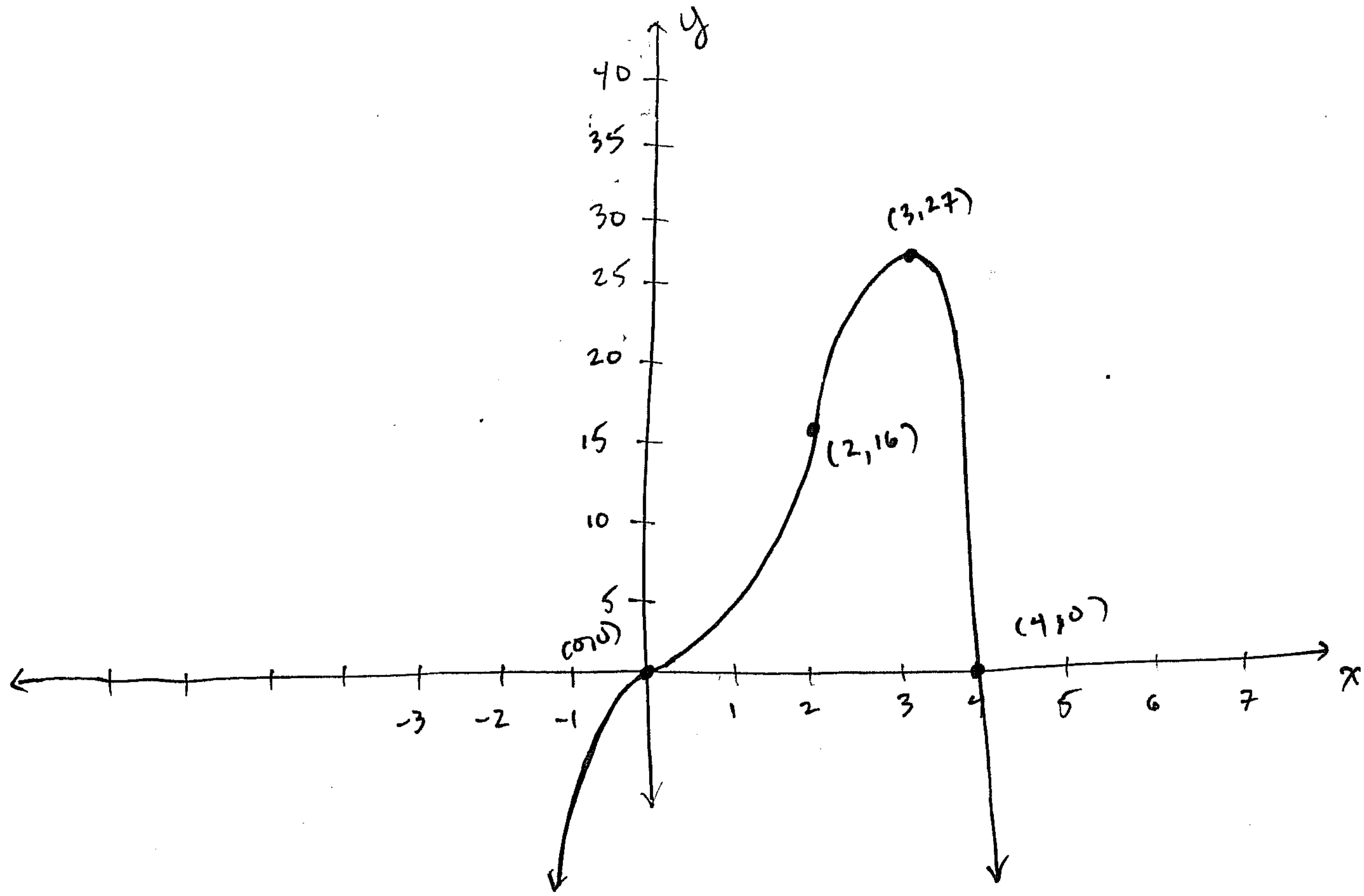
$\therefore$ f IS CONCAVE UPWARD ON $(0, 2)$
f IS CONCAVE DOWNWARD ON $(-\infty, 0)$ AND $(2, \infty)$

(f) *(1 marks)* Any inflection points.

$f(0) = 0 \Rightarrow (0, 0)$ IS AN I.P.

$f(2) = 16 \Rightarrow (2, 16)$ IS AN I.P.

(f) *(4 marks)* Sketch the graph of $f(x)$



Question 7. *(4 marks)* Use the method devoloped in class for optimizing a function on a closed interval to find the absolute maximum and absolute minimum value of $f(x) = x^3 + x^2 - 8x + 2$ on the interval $[-3, 1]$.

$$f'(x) = 3x^2 + 2x - 8 = 0$$
$$= (x+2)(3x-4) = 0$$
$$\therefore x = -2, \cancel{4/3}$$

NOT IN DOMAIN.

$$3x^2 + 6x - 4x - 8$$
$$3x(x+2) - 4(x+2)$$
$$(x+2)(3x-4)$$

C. N.

$$f(-2) = (-2)^3 + (-2)^2 - 8(-2) + 2 = 14$$

END POINTS

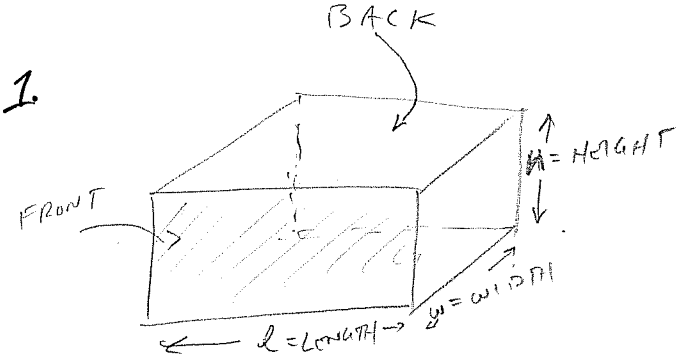
$$f(-3) = (-3)^3 + (-3)^2 - 8(-3) + 2 = 8$$
$$f(1) = (1)^3 + (1)^2 - 8(1) + 2 = -4$$

$\therefore$ $f(-2) = 14$ IS THE ABSOLUTE MAXIMUM

$f(1) = -4$ IS THE ABSOLUTE MINIMUM.

Question 8. *(6 marks)* The cabinet that will enclose the Acrosonic model D loudspeaker system will be rectangular and will have an internal volume of 2.4 ft^3. For aesthetic reasons, it has been decided that the height of the cabinet is to be 1.5 times it's width. If the top, bottom and sides of the cabinets are constructed of veneer costing \$0.40 per square foot and the front (ignoring any cutouts) and rear are constructed of particle board costing \$0.20 per square foot, what are the dimensions of the encloure that can be constructed at a minimum cost?

1.



2. $C = 2\cdot(\text{FRONT AREA})\cdot(0.20) + 2(\text{SIDE AREA})(0.40) + 2(\text{TOP AREA})(0.40)$

$= 2(l\cdot h)\,0.2 + 2wh\,(0.4) + 2(lw)\,0.4$

$= 0.4\,lh + 0.8\,wh + 0.8\,wl$

3. $h = 1.5w$

$\therefore\ C = .4\,l\cdot(1.5w) + .8\,w(1.5w) + 0.8\,wl$

$= 0.6\,wl + 1.2\,w^2 + 0.8\,wl$

$= 1.4\,wl + 1.2\,w^2$

NOW $\quad l\cdot w\cdot h = l\cdot w\cdot(1.5w) = 2.4 \Rightarrow 1.5\,lw^2 = 2.4$

$\Rightarrow l = \dfrac{2.4}{1.5w^2} = \dfrac{1.6}{w^2}$

$\therefore\ C(w) = 1.4w\left(\dfrac{1.6}{w^2}\right) + 1.2w^2 = \dfrac{2.24}{w} + 1.2w^2$

RESTRICTIONS ON w: $w > 0$ ($w \neq 0$ SINCE VOLUME $= 2.4\,ft^3$)

$\therefore$ DOMAIN OF C IS $(0, \infty)$

$C'(w) = -\dfrac{2.24}{w^2} + 2.4w = \dfrac{-2.24 + 2.4w^3}{w^2}$

$C'(w) = 0$

$-2.24 + 2.4w^3 = 0$

$\Rightarrow w = \sqrt[3]{\dfrac{2.24}{2.4}} = 0.977$

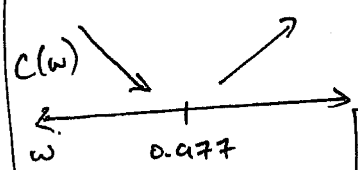
$C'(w)$ D.N.E.

$w^2 = 0$

$w = 0$

BUT THIS IS NOT IN THE DOMAIN

NOW



$C'(0.5) < 0$

$\therefore\ w = 0.977$ YIELDS ABS. MIN.

DIMENSIONS

$\therefore\ w = 0.977\,ft$

$l = 1.675\,ft$

$h = 1.4955\,ft$